Intro to Contemporary Math

Weighted Graphs and Shortest Paths

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Agenda

- Weighted Graphs
- Paths and Circuits
- Algorithms

Announcements

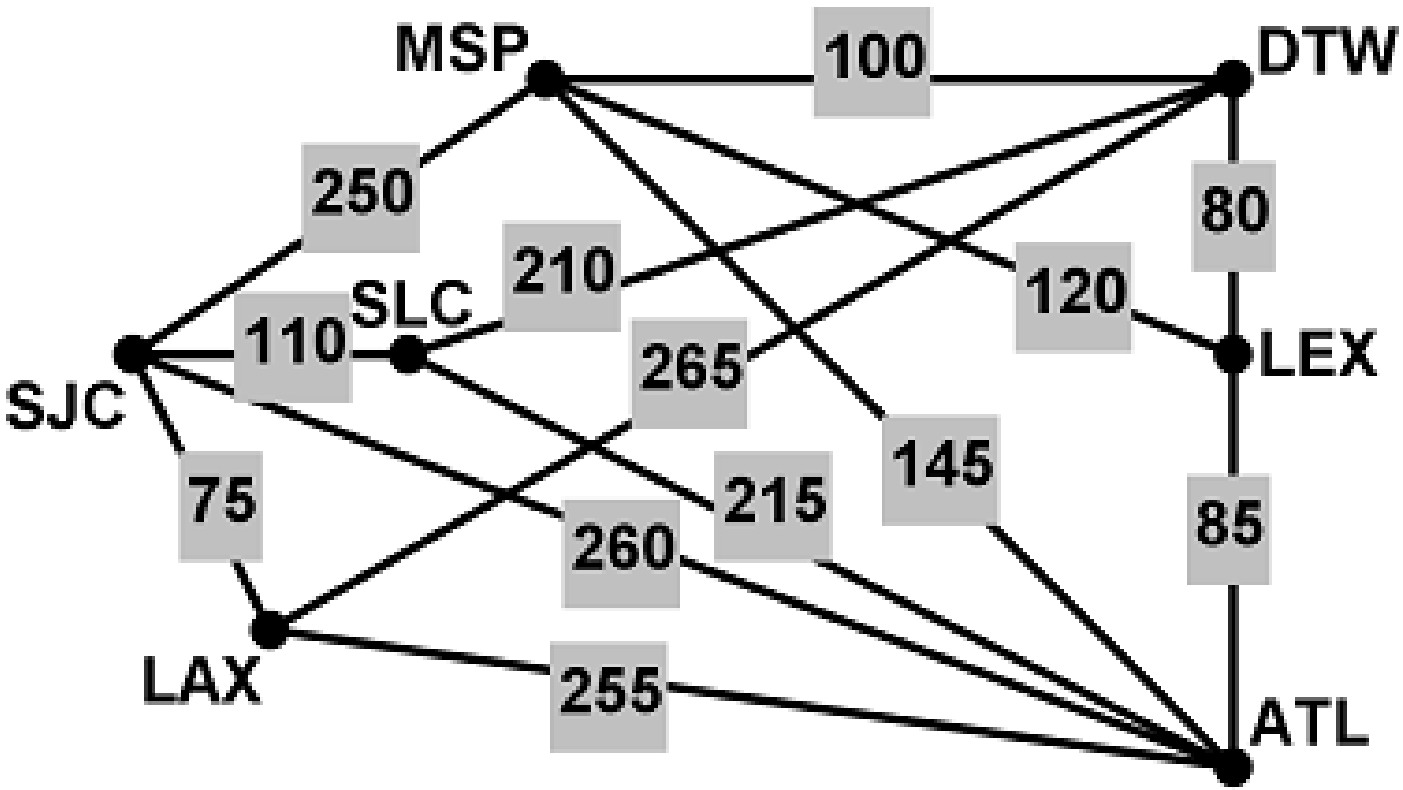
- There is a homework assignment on WebWork due tonight.
- Mini-Exam 4 is this Wednesday.

Weights

A **weight** on an edge is a numerical label representing a measurement involving the relationships modeled by the graph and its edges.

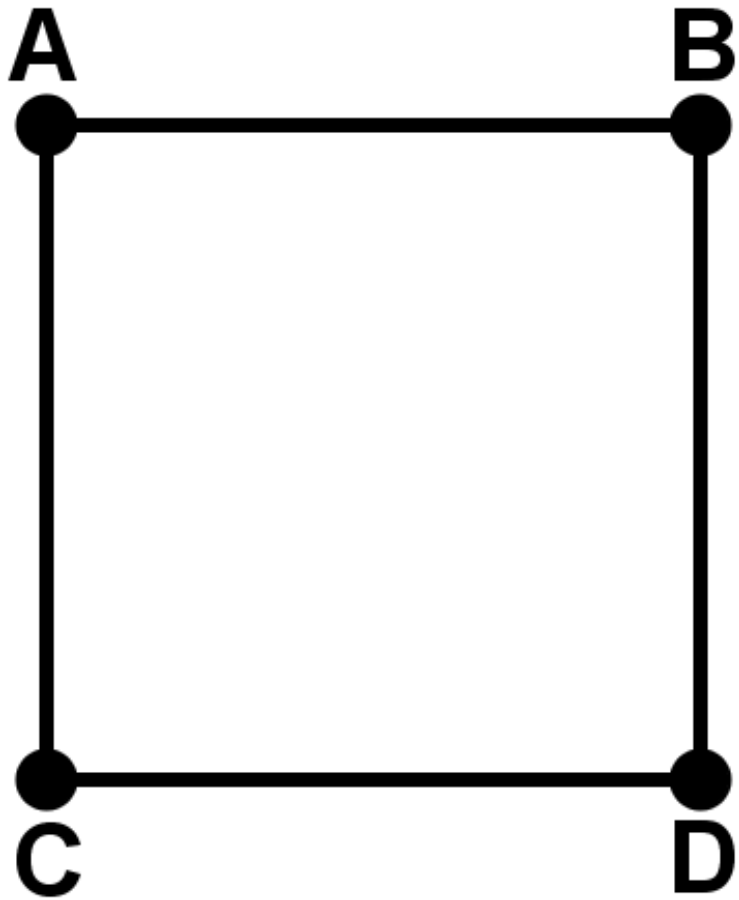
(Don't Copy) Example: Airline Routes

Vertices: Airports
Edges: Direct Flights
Weights: **flight time (minutes)**



Example

Vertices: Locations
Edges: Streets

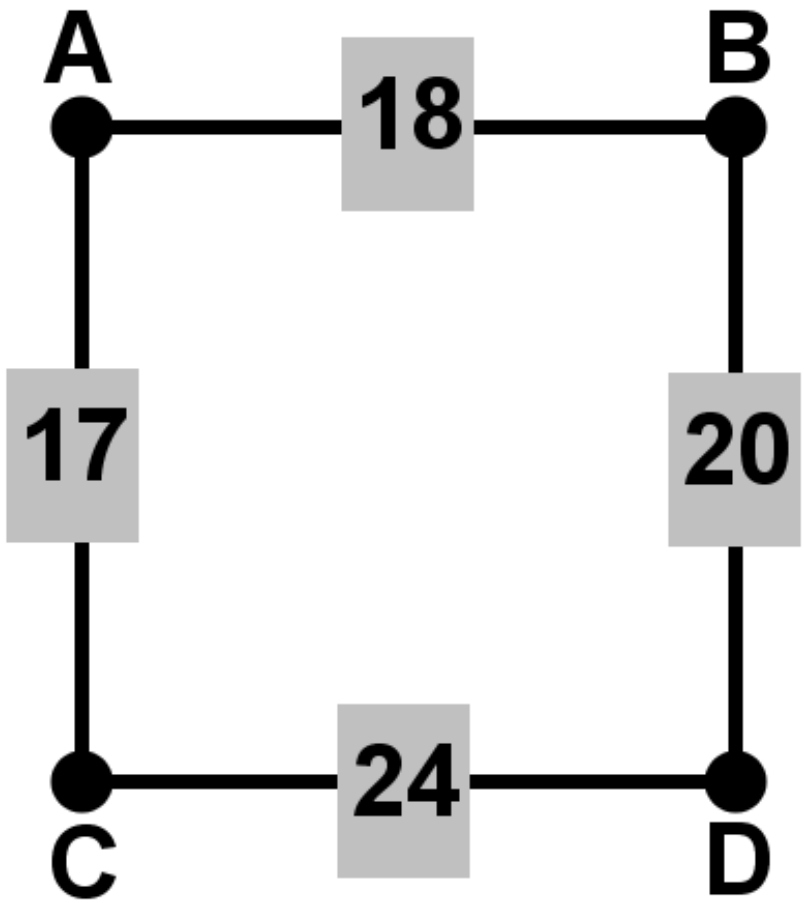


Example

Vertices: Locations

Edges: Streets

Weights: Walking Time (minutes)

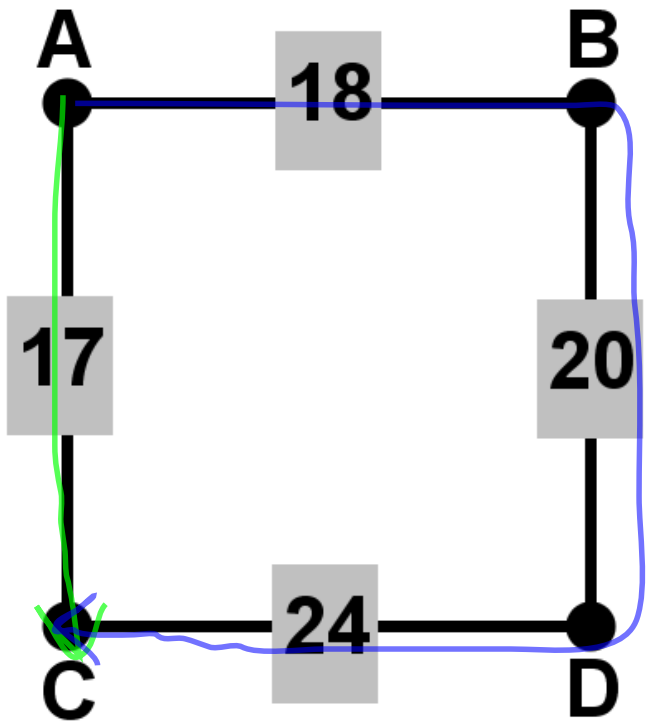


Paths and Circuits

A **path** is a sequence of vertices using the edges. For example, I can get from A to C via the path:

A, B, D, C

or just going A, C.

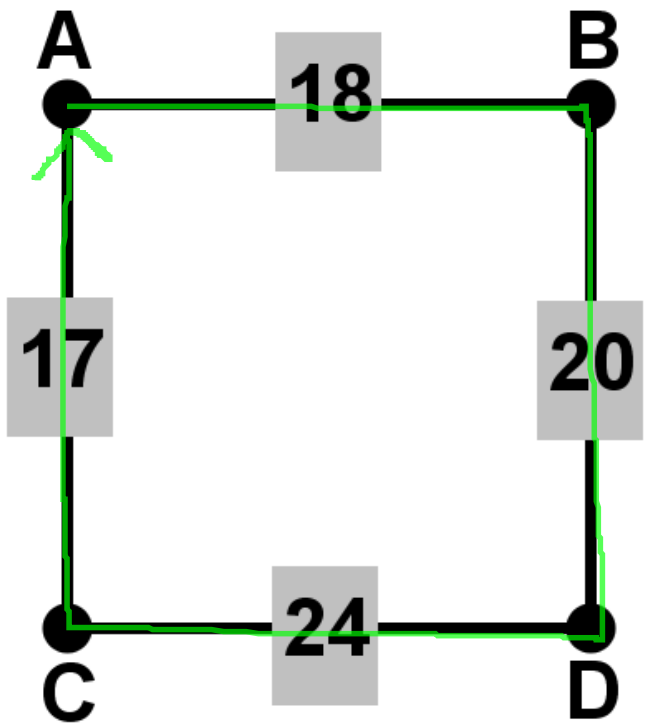


Paths and Circuits

A **circuit** is a path that starts and ends at the same vertex.

For example, a round trip from A could be

A, B, D, C, A

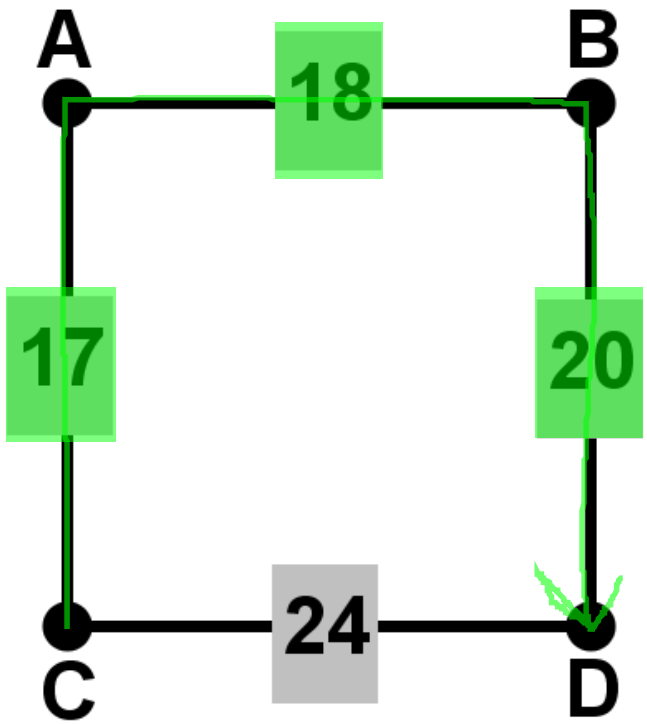


Total Weight

The **total weight** of a path is the **sum of the weights on each edge that is traveled along the path.**

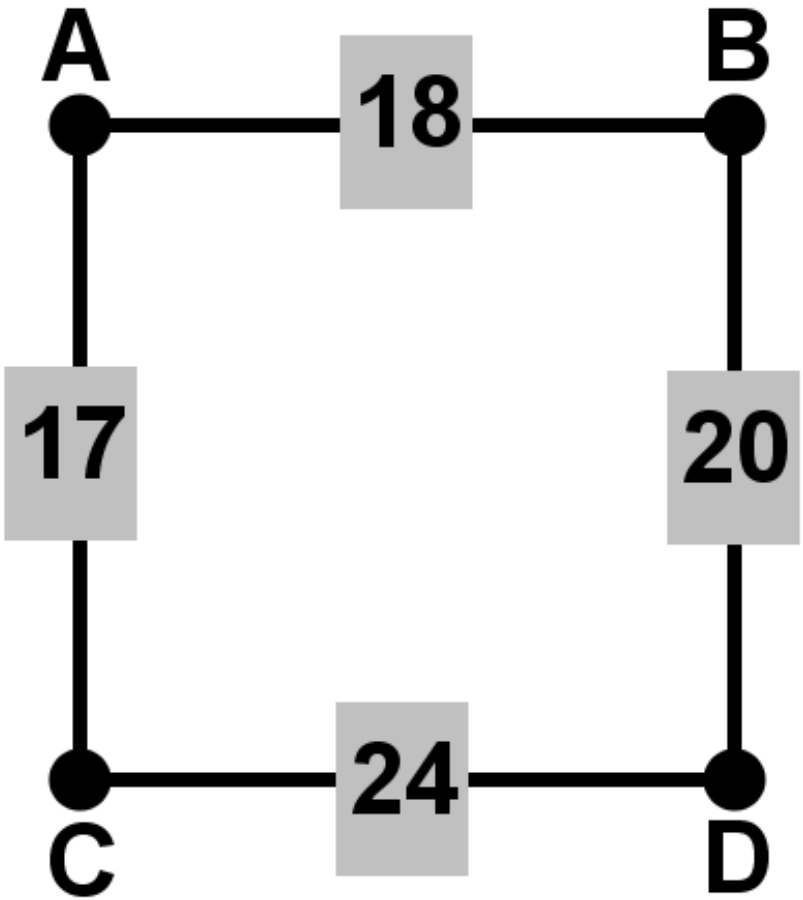
Example: The path C, A, B, D has total weight

$$17 + 18 + 20 = 55 \text{ (minutes)}$$



?(4.1) Total Weight

Find the total weight of the path A, C, D, B, A, B.

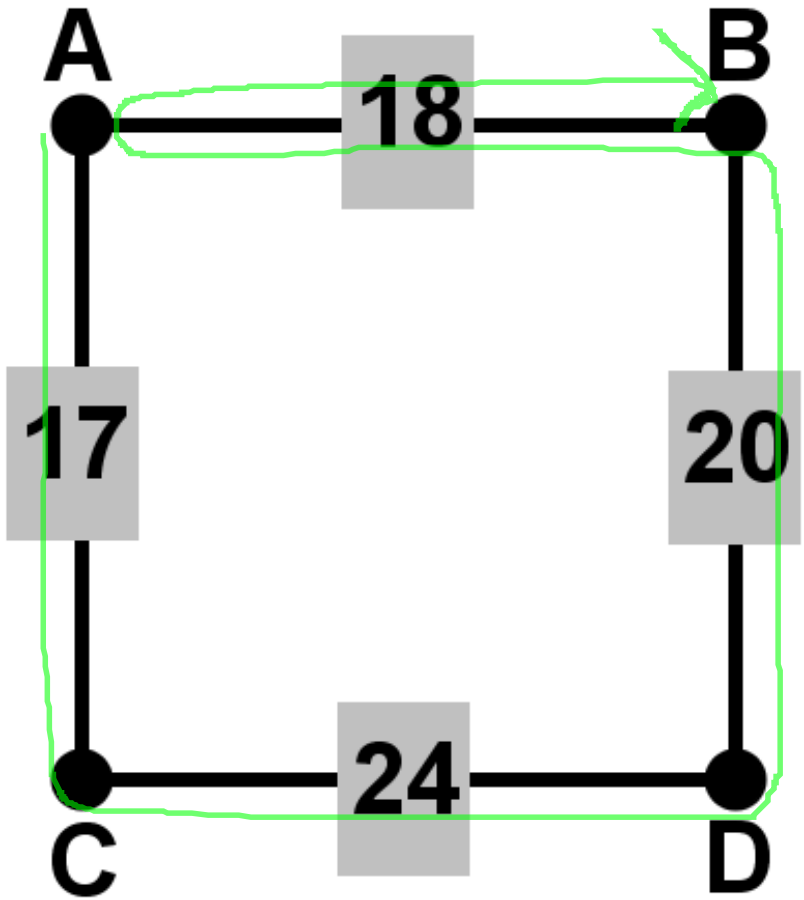


Type and send a number.

Total Weight

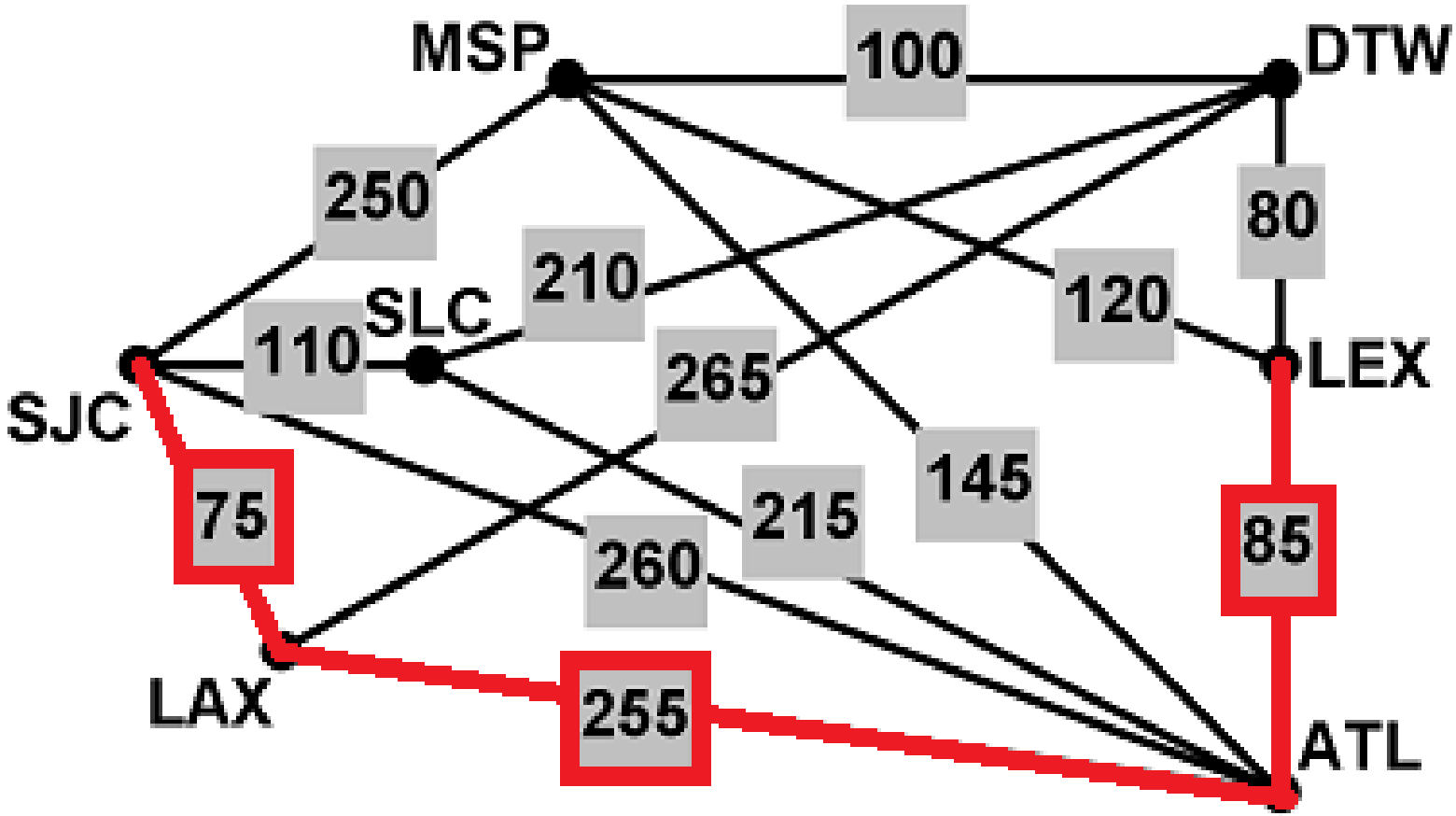
The total weight of the path A, C, D, B, A, B is

$$17 + 24 + 20 + 18 + 18 = \boxed{97}$$



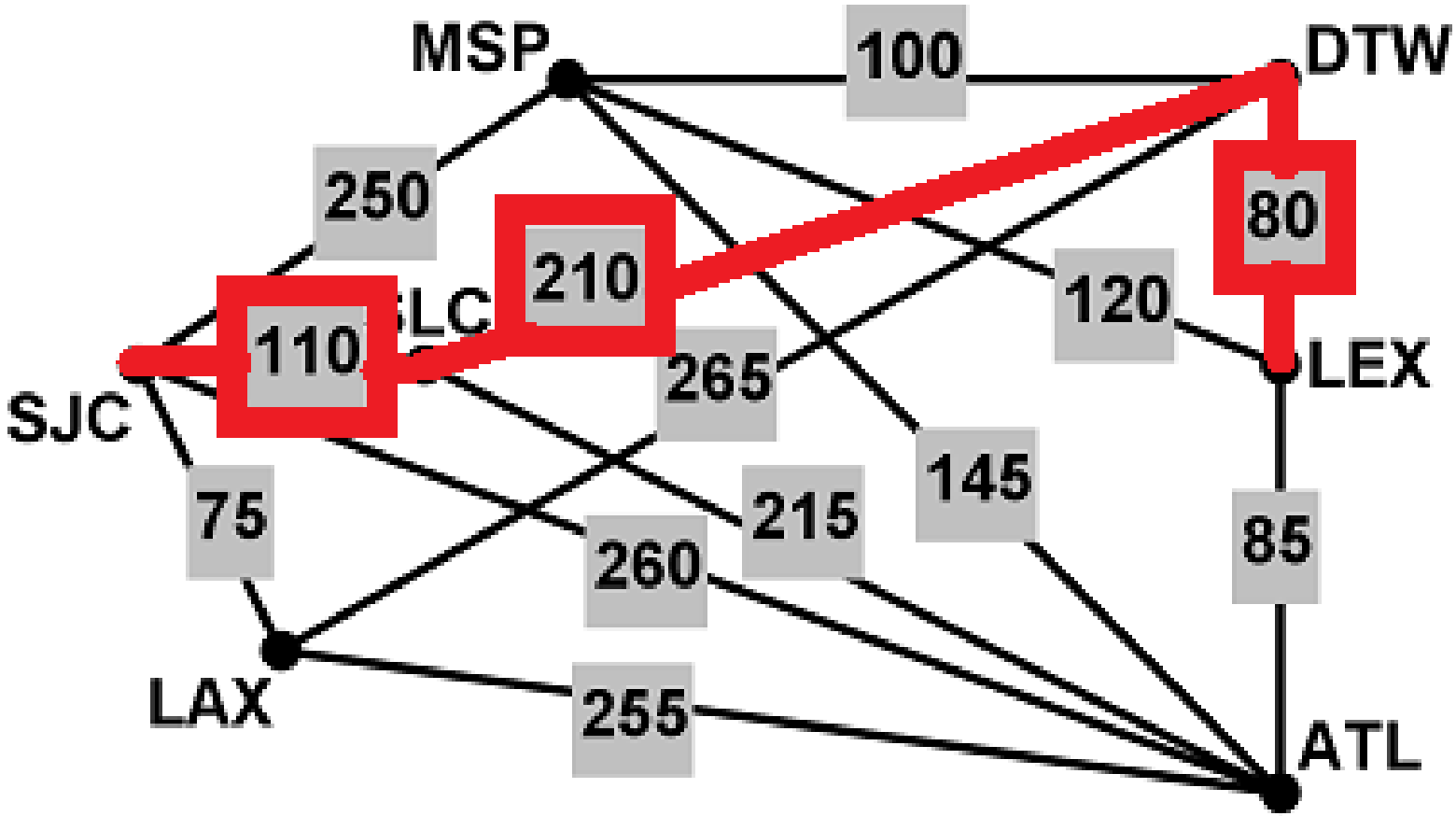
Total Weights: Flight Time

Total weight of path (LEX, ATL, LAX, SJC) is total flight time: $85 + 255 + 75 = 415$ minutes.



Total Weights: Flight Time

Total weight of path (LEX, DTW, SLC, SJC) is total flight time: $80 + 210 + 110 = 400$ minutes.

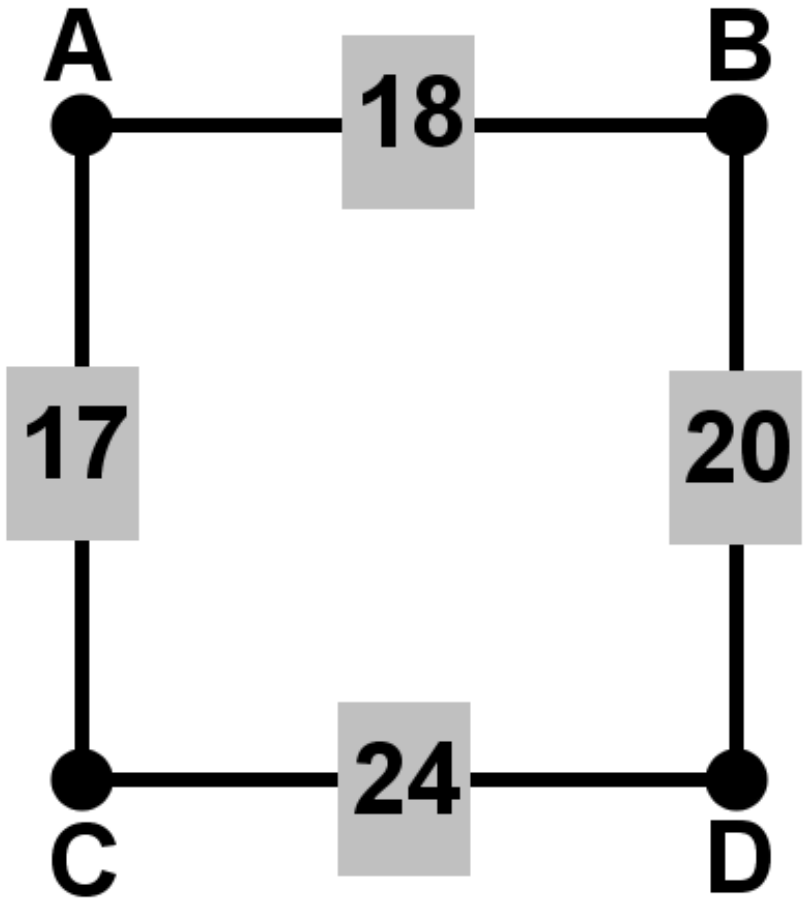


Shortest Path

The **shortest path** between two vertices A and B on a weighted graph is the path from A to B which has the lowest total weight among all paths from A to B.

?(4.2) Shortest Path

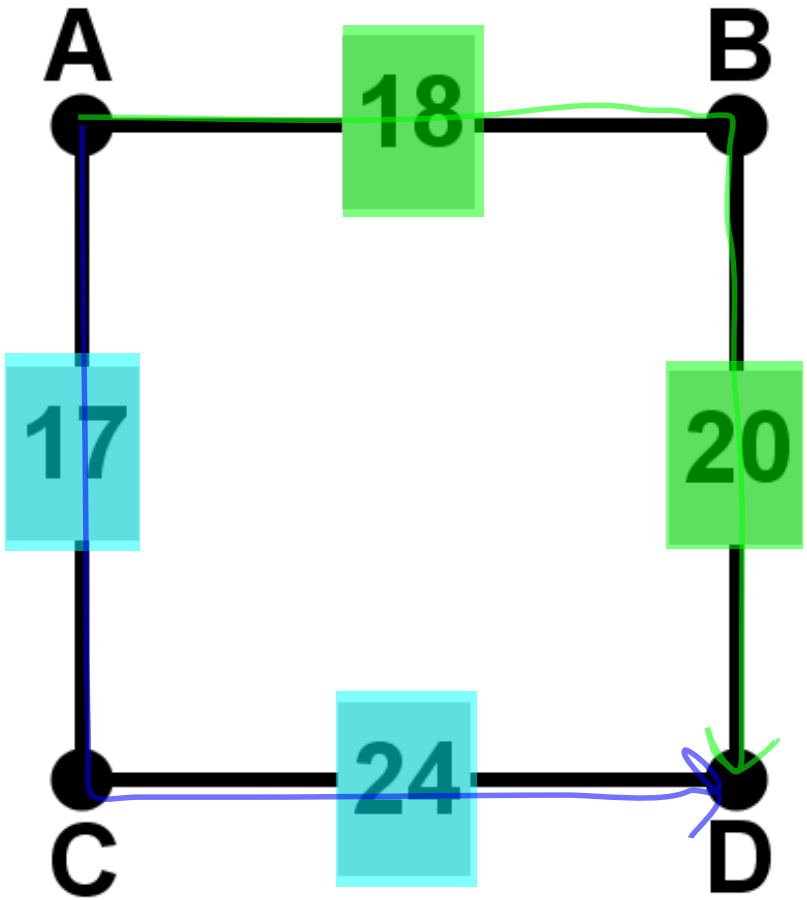
Which of these is the shortest path from A to D?



Type and send a list of vertices, starting at A and ending at D.

Shortest Path

Which of these is the shortest path from A to D?



A, B, D: $18 + 20 = 38$

A, C, D: $17 + 24 = 41$

A,B,D is the shortest path from A to D.

Algorithms

- An **algorithm** is a sequence of steps for a calculation (a procedure).
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- An **algorithm** is a sequence of steps for a calculation (a procedure).
- An **algorithm** for finding a path with the smallest total weight is **optimal** if it always produces the actual shortest path.
- An **algorithm** is **efficient** if it can be carried out in a reasonable amount of time.

Algorithm: Brute Force

- The Brute Force Algorithm searches for the shortest path between two vertices by listing every path, finding their total weights, and choosing the path with the lowest total weight.

Brute Force Evaluation

- The Brute Force Algorithm searches for the shortest path between two vertices by listing every path, finding their total weights, and choosing the path with the lowest total weight.
- The brute force algorithm is **optimal** (theoretically it will get the shortest path), but it is **inefficient** because we have to check so many possible paths.

Next Time

Dijkstra's algorithm: finding shortest paths