Intro to Contemporary Math

Compensation Intro

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Agenda

Announcement: Homework due next Monday!

- Compensation Basics
  - People and Bids
  - Types of Bids
  - Compensation and Payouts
- Fair Share
- Average Bid
- Compensation
- Examples

What is Compensation?

Compensation is the process of **awarding ownership** of some object(s) and money to people who **cannot agree on the value** of the object(s).

Example

Bob and Alice are arguing over who keeps the TV.

- Alice: item is worth 12
- Bob: item is worth 50

Compensation Basics 1.1

- Alice: item is worth 12
- Bob: item is worth 50

- The **people involved** can be described by **name or another label** (like Person 1, etc...)
- N is **how many people** are involved.

$$\text{Here, } N = 2$$

- b, decorated with a subscript of someone's name or label, is the **value/bid that person assigns to the object**.

$$b_{Alice} = 12, \; b_{Bob} = 50$$

Compensation Basics 1.2

- Alice: item is worth 12
- Bob: item is worth 50

- N is **how many people** are involved.

$$\text{Here, } N = 2$$

- b, decorated with a subscript of someone's label, is the **bid that person assigns to the object**.

$$b_{Alice} = 12, \; b_{Bob} = 50$$

- If more than one item is involved, then each person's overall bid b_{Person} is the sum of their bids on all items.

Compensation Basics 1.3

- Alice: item is worth 12
- Bob: item is worth 50

- b, decorated with a subscript of someone's label, is the **bid that person assigns to the object**.

$$b_{Alice} = 12, \; b_{Bob} = 50$$

- If more than one item is involved, then each person's overall bid b_{Person} is the sum of their bids on all items.
- The **fair share (f.s.) of a person** is the person's bid divided by the number of people: $\frac{b_{Person}}{N}$

$$\text{Alice's f.s. is } \frac{12}{2} = 6, \; \text{Bob's f.s. is } \frac{50}{2} = 25$$

Compensation Basics 1.4

- If more than one item is involved, then each person's overall bid b_{Person} is the sum of their bids on all items.
- The **fair share (f.s.) of a person** is the person's bid divided by the number of people: $\frac{b_{Person}}{N}$

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Scenarios are decided when:
1. Each Item has been given to someone
2. Cash has been exchanged

Usually, people who win items pay those who did not get them.

@Home: Fair Share

$$\frac{b_{Person}}{N}$$

Each person imagines that the items are replaced by cash equal to their bid b_{Person}. Then each of the N people can get the same amount: cash divided by number of people.
Another way to see the fair share:A person imagines that all items are replaced by a cake that is worth their own bid. If the cake is cut into N equal slices (one slice per person), the value of each slice is that person's fair share: bid (cake worth) divided by number of people.

Example Solved?

- Alice: item is worth 12
- Bob: item is worth 50

Alice wins a coin flip and gets to keep the item. She agrees to pay Bob 5.

Compensation Basics 1.5

- Alice: item is worth 12
- Bob: item is worth 50

Alice wins a coin flip and gets to keep the item. She agrees to pay Bob 5.

- Highest bid is denoted by h, and Winning bid is denoted by w.

  $h = 50$ (Bob bid more than Alice), $w = 12$ (Alice won)

Compensation Basics 1.6

Alice wins a coin flip and gets to keep the item. She agrees to pay Bob 5.

- Highest bid is denoted by h, and Winning bid is denoted by w.

  $h = 50$ (Bob bid more than Alice), $w = 12$ (Alice won)

- Warning: the winning bid does not have to be the highest bid!

Compensation Itself

Compensation: total monetary amount someone earns from the scenario. Denoted by x_{Person}. For each person,

- **Add** the person's **bids** on the items that they won (don't add bids on items others won).
- **Add/Subtract** the amount of cash that person **received/paid**.

Computing Compensation

- Alice: item is worth 12
- Bob: item is worth 50

Alice wins a coin flip and gets to keep the item. She agrees to pay Bob 5.

Bob got no item, but got cash.

$$x_{Bob} = 0 \text{ (no item)} + 5 \text{ (from Alice)} = 5$$

Computing Compensation

- Alice: item is worth 12
- Bob: item is worth 50

Alice wins a coin flip and gets to keep the item. She agrees to pay Bob 5.

Bob got no item, but got cash.

$$x_{Bob} = 0 \text{ (no item)} + 5 \text{ (from Alice)} = 5$$

Alice got the item, but paid cash.

$$x_{Alice} = 12 \text{ (her bid on item)} - 5 \text{ (to Bob)} = 7$$

Average Bid

- **Average Bid:** add up everyone's bid, then divide by the number of people N. It is denoted by m.

$$m = \frac{b_1 + b_2 + \ldots + b_N}{N}$$

The average bid is also the sum of everyone's fair shares.

?(1.1) Bid on Two Items

Alice and Bob are fighting over two items

Items	Alice's Bids	Bob' Bids
TV	200	260
desk	100	80

- What is b_{Bob}? Type and send a number.

Bid on Two Items Answers

Items	Alice's Bids	Bob's Bids
TV	200	260
desk	100	80

- Bob's bid is the sum of his bids on each item:
  $b_{Bob} = 260 + 80 = 340$

?(1.2) Car Trouble 1.1

Tim, Shawn, and **Leo** need to decide who keeps the car. Tim is willing to pay \$13,500, Shawn thinks the car is worth \$12,000, and Leo values the car at \$9,000. A neighbor picks Tim as the winner, and he decides to pay Shawn and Leo \$4,500 each.
Find w. Type and send a number.

Car Trouble 1.1

Tim, Shawn, and **Leo** need to decide who keeps the car. Tim is willing to pay \$13,500, Shawn thinks the car is worth \$12,000, and Leo values the car at \$9,000. A neighbor picks Tim as the winner, and he decides to pay Shawn and Leo \$4,500 each.

Tim's bid is the highest, so

$$h = 13{,}500, \text{ Tim's bid}$$

Tim also won, so

$$w = 13{,}500, \text{ Tim's bid}$$

?(1.3) Car Trouble 1.2

Tim, Shawn, and **Leo** need to decide who keeps the car. Tim is willing to pay $13,500, Shawn thinks the car is worth $12,000, and Leo values the car at $9,000. A neighbor picks Tim as the winner, and he decides to pay Shawn and Leo $4,500 each.

Find Shawn's fair share:

A) 3,000 B) 4,000 C) 4,500 D) 6,000 E) 12,000

Car Trouble 1.2

Tim, Shawn, and **Leo** need to decide who keeps the car. Tim is willing to pay \$13,500, Shawn thinks the car is worth \$12,000, and Leo values the car at \$9,000. A neighbor picks Tim as the winner, and he decides to pay Shawn and Leo \$4,500 each.

Shawn's fair share is his bid b_{Shawn} divided by the number of people N:

$$b_{Shawn} = 12,000, \quad N = 3,$$

$$\text{Fair share is } \frac{12,000}{3} = \boxed{4,000}$$

?(1.4) Car Trouble 1.3

Tim, Shawn, and **Leo** need to decide who keeps the car. Tim is willing to pay $13,500, Shawn thinks the car is worth $12,000, and Leo values the car at $9,000. A neighbor picks Tim as the winner, and he decides to pay Shawn and Leo $4,500 each.

What is Shawn's compensation? Remember: he did not win the item, but he got cash from Tim.

A) 3,000 B) 4,000 C) 4,500 D) 9,000 E) 12,000 F) 16,500

Car Trouble 1.3

Tim, Shawn, and **Leo** need to decide who keeps the car. Tim is willing to pay \$13,500, Shawn thinks the car is worth \$12,000, and Leo values the car at \$9,000. A neighbor picks Tim as the winner, and he decides to pay Shawn and Leo \$4,500 each.

Shawn got no item, but got cash from Tim, so

$$x_{Shawn} = 0 + 4,500 = \boxed{4,500}$$

?(1.5) Car Trouble 1.4

Tim, Shawn, and **Leo** need to decide who keeps the car. Tim is willing to pay \$13,500, Shawn thinks the car is worth \$12,000, and Leo values the car at \$9,000. A neighbor picks Tim as the winner, and he decides to pay Shawn and Leo \$4,500 each.

What is Tim's compensation?

A) $-9{,}000$ B) 4,500 C) 9,000 D) 13,500 E) 18,000 F) 22,500

Car Trouble 1.4

Tim, Shawn, and **Leo** need to decide who keeps the car. Tim is willing to pay \$13,500, Shawn thinks the car is worth \$12,000, and Leo values the car at \$9,000. A neighbor picks Tim as the winner, and he decides to pay Shawn and Leo \$4,500 each.

Tim won, so he gets his bid on the item, but he paid 4,500 to Shawn and to Leo, so

$$x_{Tim} = \underbrace{13,500}_{\text{ITEM}} - \underbrace{4,500}_{\text{PAY SHAWN}} - \underbrace{4,500}_{\text{PAY LEO}} = 4,500$$

@Home: Car Trouble Full

Tim, Shawn, and **Leo** need to decide who keeps the car. Tim is willing to pay \$13,500, Shawn thinks the car is worth \$12,000, and Leo values the car at \$9,000. A neighbor picks Tim as the winner, and he decides to pay Shawn and Leo \$4,500 each.
Find:

- N
- b_{Shawn}
- b_{Leo}
- b_{Tim}
- h
- w
- m
- Tim's f.s.
- Shawn's f.s.
- Leo's f.s.
- x_{Shawn}
- x_{Leo}
- x_{Tim}

Car Trouble Full Answers

- $N = 3$
- $b_{Shawn} = 12,000$
- $b_{Leo} = 9,000$
- $b_{Tim} = 13,500$
- $h = 13,500$
- $w = 13,500$
- $m = \frac{13,500+12,000+9,000}{3}$
  $m = 11,500$

- T's f.s. is $\frac{13,500}{3} = 4,500$
- S's f.s. is $\frac{12,000}{3} = 4,000$
- L's f.s. is $\frac{9,000}{3} = 3,000$
- $x_{Shawn} = 4,500$
- $x_{Leo} = 4,500$

- $x_{Tim} = \underbrace{13,500}_{\text{ITEM}} - \underbrace{4,500}_{\text{PAY SHAWN}} - \underbrace{4,500}_{\text{PAY LEO}} = 4,500$

Next time

- A procedure that guarantees everyone getting their fair share... and more!