Intro to Contemporary Math

Knaster's Method (part one)

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Agenda

- Project Announcement
- Knaster's Method

WebWork

- You have an assignment due tonight.
- There will be homework posted on Wednesday. It will cover up to this Friday's material. It will be due next Monday.

Knaster's Procedure

This procedure focuses on giving all participants their fair share, and possibly more.

Items	Alice's Bids	Bob's Bids
TV	200	260
desk	100	80

1) Make a **Table:**

People	Bids	F. Share	Items	Value	1st $
Alice					
Bob					

Knaster's Procedure Part One

People	Bids	F. Share	Items	Value	1st $
Alice					
Bob					

2) Compute **everyone's (overall) bids**
3) Compute **everyone's fair share**
4) Determine **who won each item.** The winner of each item **bid the most on that item.**
5) For each person, **add up their bids** on the **items they won.** This is their **value** of the items they won.
6) For each person, compute:

$$\text{Fair share} - \text{Value of Items Won} = \text{1st \$}$$

This is each person's **first settlement (cashflow).** It tells us who needs more money and who needs to pay.

- If 1st $ is positive, the person is getting cash.
- If 1st $ is negative, the person is paying cash.

Knaster's Procedure Part Two

- Leftover money is called the **surplus**. In general, compute surplus by:
  1) Adding up numbers in (1st \$) column (watch signs!)
  2) Multiplying by -1.
  Surplus must be **positive**!
- The mediator distributes the surplus as follows: each person gets

$$\left(\frac{\text{Amount of surplus}}{\text{Number of people}}\right) \text{ dollars}$$

  from the surplus. This is added to their 1st settlements to get **final settlements** (overall cashflow).
  - People who received money can get more from surplus.
  - People who paid earlier can get some money back.

Knaster's Procedure Compensation

People	Surplus	Final $	Comp.
Alice			
Bob			

Compensation x_{Person} is total monetary value from items and cash:

- **Add** the **value** of items that the person won
- **Add/Subtract** the amount of cash that person **received/paid.**

In Knaster's method, the value of items won is recorded in "Value," and the amount of cash is recorded in "Final $."

Knaster's Procedure

This procedure focuses on giving all participants their fair share, and possibly more. A mediator gets involved to handle the exchange of money.
Hint: Start a new page in your notes.

Items	Alice's Bids	Bob's Bids
TV	200	260
desk	100	80

1) Make a **Table:**

People	Bids	F. Share	Items	Value	1st $
Alice					
Bob					

Knaster's Procedure Step 2

2) Compute **everyone's (overall) bids:**

Items	Alice's Bids	Bob's Bids
TV	200	260
desk	100	80

Alice's bid is $b_{Alice} = 200 + 100 = 300$
Bob's bid is $b_{Bob} = 260 + 80 = 340$

People	Bids	F. Share	Items	Value	1st $
Alice	300				
Bob	340				

Knaster's Procedure Step 3

Alice's bid is $b_{Alice} = 200 + 100 = 300$
Bob's bid is $b_{Bob} = 260 + 80 = 340$
3) Compute **everyone's fair share**

- Alice's fair share is $\frac{b_{Alice}}{2} = \frac{300}{2} = 150$
- Bob' fair share is $\frac{b_{Bob}}{2} = \frac{340}{2} = 170$

Divide by 2 because there are 2 people

People	Bids	F. Share	Items	Value	1st $
Alice	300	150			
Bob	340	170			

?(2.2) Knaster's Procedure Step 4

4) Determine **who won each item.** The winner of each item **bid the most on that item.**

Items	Alice's Bids	Bob's Bids
TV	200	260
desk	100	80

1) Alice won:
2) Bob won:

A) TV
B) desk

Type and send two letters.

Knaster's Procedure Step 4

4) Determine **who won each item.** The winner of each item **bid the most on that item.**

Items	Alice's Bids	Bob's Bids
TV	200	260
desk	100	80

- Alice won the desk, while Bob won the TV.

People	Bids	F. Share	Items	Value	1st $
Alice	300	150	desk		
Bob	340	170	TV		

?(2.3) Knaster's Procedure Step 5

5) For each person, **add up their bids** on the **items they won.** This is their **value** of the items they won.

Items	Alice's Bids	Bob's Bids
TV	200	260
desk	100	80

1) Alice's value
2) Bob's value

A) 80
B) 100
C) 200
D) 260
E) 300
F) 340

Type and send two letters.

Knaster's Procedure Step 5

5) For each person, **add up their bids** on the **items they won.** This is their **value** of the items they won.

Items	Alice's Bids	Bob's Bids
TV	200	**260**
desk	**100**	80

- Alice won the desk. Her items are worth a total of **100** to her.
- Bob won the TV. His items are worth a total of **260** to him.

Knaster's Procedure Step 5 Table

Fill in table:

People	Bids	F. Share	Items	Value	1st $
Alice	300	150	desk	100	
Bob	340	170	TV	260	

At this point,

- Alice got 100 in value, **but she wanted at least** 150 (her fair share).
- Bob got 260 in value, **more** than his fair share of 170.

Cash will be exchanged to fix this.

Knaster's Procedure Step 6

6) For each person, compute:

$$\text{Fair share} - \text{Value of Items Won} = \text{1st \$}$$

This is each person's **first settlement (cashflow)**. It tells us who needs more money and who needs to pay.

- If 1st \$ is positive, the person is getting cash.
- If 1st \$ is negative, the person is paying cash.

Knaster's Procedure Step 6

$$\text{Fair share} - \text{Value of Items Won} = \text{1st \$}$$

This is each person's **first settlement (cashflow)**. It tells us who needs more money and who needs to pay.

People	Bids	F. Share	Items	Value	1st \$
Alice	300	150	desk	100	50
Bob	340	170	TV	260	

Alice's 1st \$ is

$$150 - 100 = \boxed{50}$$

Alice's first settlement is **positive**, because she **got less than her fair share**. She needs to **get \$50**.

?(2.4) Knaster's Procedure Step 6

People	Bids	F. Share	Items	Value	1st $
Alice	300	150	desk	100	50
Bob	340	170	TV	260	

Alice's 1st $ is

$$150 - 100 = \boxed{50}$$

Alice's first settlement is **positive**, because she **got less than her fair share**. She needs to **get $50**.
What is Bob's 1st $?

A) 80
B) -80
C) 90
D) -90
E) 170
F) -170

Knaster's Procedure Step 6

People	Bids	F. Share	Items	Value	1st $
Alice	300	150	desk	100	50
Bob	340	170	TV	260	−90

Bob's 1st $ is

$$170 - 260 = \boxed{-90}$$

Bob's first settlement is **negative**, because he **got more than his fair share**. He will need to **pay $90**.

Knaster's Procedure Step 6

- People who pay will give their money to a common pool (the "pot"); the others will then take money from it.
  Now everyone has their fair share!
  - Bob puts \$90 into pot; Alice takes \$50 from it.

Knaster's Procedure Surplus

- People who pay will give their money to a common pool (the "pot"); the others will then take money from it.
  Now everyone has their fair share!
  - Bob puts \$90 into pot; Alice takes \$50 from it.
- Notice that pot still has \$40. Leftover money is called the **surplus**. In general, compute surplus by:
  1) Adding up numbers in (1st \$) column (watch signs!)
  2) Multiplying by -1.
  Surplus must be **positive**!

Knaster's Procedure Surplus Calculation

People	Bids	F. Share	Items	Value	1st \$
Alice	300	150	desk	100	50
Bob	340	170	TV	260	-90

- Notice that pot still has \$40. Leftover money is called the **surplus**. In general, compute surplus by:
  1) Adding up numbers in (1st \$) column (watch signs!)
  2) Multiplying by -1.
  Surplus must be **positive**!

$$(50 - 90) \times (-1) = 40$$

The surplus in this scenario is \$40.

Knaster's Procedure Surplus Calculation

The surplus in this scenario is \$40.

- The mediator distributes the surplus as follows: each person gets

$$\left(\frac{\text{Amount of surplus}}{\text{Number of people}}\right) \text{ dollars}$$

  from the surplus. This is added to their 1st settlements to get **final settlements** (overall cashflow).
- People who received money can get more from surplus.
- People who paid earlier can get some money back.

Knaster's Procedure Blank Tables

People	Bids	F. Share	Items	Value	1st \$
Person 1					
Person 2					
...					
Person N					

People	Value	1st \$	Surplus	Final \$	Comp.
Person 1					
Person 2					
...					
Person N					

@Home: Example with Three People, Three Items

	Annie	Corinna	Florence
Fan	21	12	18
Heater	51	24	36
Cooker	15	45	30

People	Bids	F. Share	Items	Value	1st $
Annie					
Corinna					
Florence					

@Home: Example with Three People, Three Items, Steps 1-3

	Annie	Corinna	Florence
Fan	21	12	18
Heater	51	24	36
Cooker	15	45	30

$$b_{Annie} = 21 + 51 + 15 = 87, \text{ Fair Share } \frac{87}{3} = 29$$

$$b_{Corinna} = 12 + 24 + 45 = 81, \text{ Fair Share } \frac{81}{3} = 27$$

$$b_{Florence} = 18 + 36 + 30 = 84, \text{ Fair Share } \frac{84}{3} = 28$$

@Home: Example with Three People, Three Items, Steps 1-3

People	Bids	F. Share	Items	Value	1st $
Annie	87	29			
Corinna	81	27			
Florence	84	28			

@Home: Example with Three People, Three Items, Steps 4-5

	Annie	Corinna	Florence
Fan	21	12	18
Heater	51	24	36
Cooker	15	45	30

- Annie wins Fan and Heater, total value $21 + 51 = 72$
- Corinna wins Cooker, total value 45
- Florence wins nothing, total value 0

@Home: Example with Three People, Three Items, Steps 4-5

People	Bids	F. Share	Items	Value	1st $
Annie	87	29	Heater, Fan	72	
Corinna	81	27	Cooker	45	
Florence	84	28	None	0	

@Home: Example with Three People, Three Items, Step 6

People	Bids	F. Share	Items	Value	1st \$
Annie	87	29	Heater, Fan	72	-43
Corinna	81	27	Cooker	45	-18
Florence	84	28	None	0	28

- Annie \$: $29 - 72 = -43$ (pay)
- Corinna \$: $27 - 45 = -18$ (pay)
- Florence \$: $28 - 0 = 28$ (get)
- Pot has $-1 \times (-43 - 18 + 28) = 33$ (surplus)

@Home: Example with Three People, Three Items, Final

People	Value	1st \$	Surplus	Final \$	Comp.
Annie	72	-43	$+11$	-32	40
Corinna	45	-18	$+11$	-7	38
Florence	0	28	$+11$	39	39

- Pot has \$33, so everyone gets

$$\frac{33}{3} = 11 \text{ dollars}$$

- Annie gets $x_{Annie} = 72 - 32 = 40$
- Corinna gets $x_{Corinna} = 45 - 7 = 38$
- Florence gets $x_{Florence} = 0 + 39 = 39$

@Home Example where a Winner gets Paid

	Annie	Corinna	Florence
Fan	18	12	21
Heater	51	24	36
Cooker	15	45	30

People	Bids	F. Share	Items	Value	1st $
Annie	84	28	Heater	51	−23
Corinna	81	27	Cooker	45	−18
Florence	87	29	Fan	21	8

@Home Example where a Winner gets Paid

- Pot has $-1 \times (-23 - 18 + 8) = 33$ dollars, so everyone gets $\frac{33}{3} = 11$ dollars:

People	Value	1st \$	Surplus	Final \$	Comp.
Annie	51	-23	$+11$	-12	39
Corinna	45	-18	$+11$	-7	38
Florence	21	8	$+11$	19	40

- Annie gets $x_{Annie} = 51 - 12 = 39$
- Corinna gets $x_{Corinna} = 45 - 7 = 38$
- Florence gets $x_{Florence} = 21 + 19 = 40$

Next time

- More practice
- Return on Bids: XB ratios