Intro to Contemporary Math

Adjusted Winner Procedure

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Agenda

- Adjusted Winner Method
  - Point Spending
  - Point Ratios
  - Transfer Equations

Announcements

- Homework due tonight
- Mini-Exam on Wednesday

Adjusted Winner Main Idea

Two people are given 100 points to spend on their bids.
At first, the **highest bidder** on each item wins that item.
Then, items are given to the other person or shared so that both people get **the same number of points from items won.**

Adjusted Winner Steps 1-2

1) Each person **makes their bids:**

	Alice	Bob	Point Ratio
Ring	45	40	
Desk	21	20	
TV	5	20	
Sofa	29	20	
Points from Items Won			

2) The **highest bidder** on each item wins that item. Add up each person's **points from the items they won.**

Point Ratio and Steps 3-5

3) Compute the **point ratios** of each item. The **point ratio** for each item is

$$\frac{\text{Points bid by } \textbf{Winner} \text{ of Item}}{\text{Points bid by } \textbf{Loser} \text{ of Item}}$$

4) The person who **has more points will give items** to the other person. This person is the **Giver**.
5) Transfer the **Giver's item with the lowest point ratio** to the other person. Adjust the Points from Items Won.
If the Giver ends up with fewer points than the other person, cancel the transfer.
Repeat Step 5 as needed until a transfer is cancelled.

Step 6

6) If the Giver still has more points than the other person but cannot give any items, the two people must **share the item.**

- To **share an item** means that each person will **have a fraction (between 0 to 1)** of the item.
- The sum of the amount owned by both people must add up to 1.
- Each person gets back that fraction of their bid on the shared item.
- Goal: Split item so that both people have same amount of points.

Transfer Equation and Step 7

7) Write the transfer equation.
Let s be the fraction of a shared item owned by the Giver. We find s by solving the **transfer equation:**

$$\begin{aligned} & s \times (\text{Giver's bid on Shared item}) \\ & + (\text{Giver's points minus bid on the Shared item}) \\ = & (1-s) \times (\text{Other Person's Bid on Shared Item}) \\ & + (\text{Other person's points}) \end{aligned}$$

Transfer Equation Details

- $s \times$ (Giver's Bid on Shared Item)
  - Giver gets back this fraction of bid on shared item.
- (Giver's points minus bid on the Shared item)
  - This is how many points the Giver has. When sharing the item, we take it from the giver for a moment.
- $(1 - s) \times$ (Other Person's Bid on Shared Item)
  - Other person gets remaining $1 - s$ fraction of shared item, and gets back this fraction of bid.
- (Other person's points)
  - This is how many points the other person has has so far.
- Remember, we want both people to have has the same amount of points, so we share an item to accomplish this.

Finish (Step 8)

8) Solve the transfer equation. This tells us the portion (out of 1) of the shared item that each person receives.

- The number of points from each person's portion of the item is their portion (as a fraction or decimal) multiplied by their original bid on the shared item.
- Upon knowing this, the compensation scenario is resolved. We know what items (or portions of items) each person has, and the number of points they get from their items.

Adjusted Winner Procedure Step 1

1) Each person **makes their bids:**

	Alice	Bob	Point Ratio
Ring	45	40	
Desk	21	20	
TV	5	20	
Sofa	29	20	
Points from Items Won			

Adjusted Winner Procedure Step 2

2) The **highest bidder** on each item wins that item. Add up each person's **points from the items they won:**

	Alice	Bob	Point Ratio
Ring	**45**	40	
Desk	**21**	20	
TV	5	**20**	
Sofa	**29**	20	
Points from Items Won	**95**	**20**	

2) The **highest bidder** on each item wins that item. Add up each person's **points from the items they won:**

	Alice	Bob	Point Ratio
Ring	**45**	40	
Desk	**21**	20	
TV	5	**20**	
Sofa	**29**	20	
Points from Items Won	**95**	**20**	

- Alice has $45 + 21 + 29 = 95$
- Bob has $20 = 20$

Point Ratio

The **point ratio** for each item is

$$\frac{\text{Points bid by } \mathbf{Winner} \text{ of Item}}{\text{Points bid by } \mathbf{Loser} \text{ of Item}}$$

Point Ratio

The **point ratio** for each item is

$$\frac{\text{Points bid by } \textbf{Winner} \text{ of Item}}{\text{Points bid by } \textbf{Loser} \text{ of Item}}$$

	Alice	Bob
Ring	**45**	40

Point ratio for Ring:

$$\frac{45}{40} = 1.125$$

?(5.1) Adjusted Winner Procedure Step 3

$$\textit{Ring}: \quad \frac{45}{40} = 1.125 \qquad\qquad \textit{Desk}: \quad \frac{21}{20} = 1.05$$

	Alice	Bob	Point Ratio
Ring	**45**	40	1.125
Desk	**21**	20	1.05
TV	5	**20**	
Sofa	**29**	20	
Points from Items Won	**95**	**20**	

What number is the point ratio for the TV? Type and send a number.

3) Compute the point ratios of each item

Ring:

$$\frac{45}{40} = 1.125$$

Desk:

$$\frac{21}{20} = 1.05$$

TV:

$$\frac{20}{5} = 4$$

Sofa:

$$\frac{29}{20} = 1.45$$

Watch out on the TV: Bob won it, so his bid goes on top.

Adjusted Winner Procedure Step 4

4) The person who **has more points will give items** to the other person. This person is the **Giver**.

	Alice	Bob	Point Ratio
Ring	**45**	40	1.125
Desk	**21**	20	1.05
TV	5	**20**	4
Sofa	**29**	20	1.45
Points from Items Won	**95**	**20**	

- Alice has more points than Bob, so she's the **Giver**.

Adjusted Winner Procedure Step 5

5) Transfer the **Giver's item with the lowest point ratio** to the other person. Adjust the Points from Items Won.
If the Giver ends up with fewer points than the other person, cancel the transfer.

	Alice	Bob	Point Ratio
Ring	**45**	40	1.125
Desk	21→	**20**	1.05
TV	5	**20**	4
Sofa	**29**	20	1.45
Points from Items Won	**74**	**40**	

- Alice gives the Desk to Bob:

5) Transfer the **Giver's item with the lowest point ratio** to the other person. Adjust the Points from Items Won.
If the Giver ends up with fewer points than the other person, cancel the transfer.

	Alice	Bob	Point Ratio
Ring	**45**	40	1.125
Desk	21→	**20**	1.05
TV	5	**20**	4
Sofa	**29**	20	1.45
Points from Items Won	**74**	**40**	

- Alice gives the Desk to Bob:
  Alice loses 21 (*her bid* on Desk) and drops from 95 to **74** points
  Bob gains 20 (*his bid* on Desk) and goes up from 20 to **40** points.

	Alice	Bob	Point Ratio
Ring	**45**	40	1.125
Desk	21→	**20**	1.05
TV	5	**20**	4
Sofa	**29**	20	1.45
Points from Items Won	**74**	**40**	

- Alice gives the Desk to Bob:
  Alice loses 21 (*her bid* on Desk) and drops from 95 to **74** points
  Bob gains 20 (*his bid* on Desk) and goes up from 20 to **40** points.
  Alice still has more points.

?(5.2) Adjusted Winner Procedure Step 5, Part 2

	Alice	Bob	Point Ratio
Ring	**45**	40	1.125
Desk	21→	**20**	1.05
TV	5	**20**	4
Sofa	**29**	20	1.45
Points from Items Won	**74**	**40**	

What item should Alice **try** to transfer next? Type and send the item's name.

	Alice	Bob	Point Ratio
Ring	**45**	40	1.125
Desk	21→	**20**	1.05
TV	5	**20**	4
Sofa	**29**	20	1.45
Points from Items Won	**74**	**40**	

- Alice **tries** to gives the Ring to Bob, but:
  Alice would lose 45 (*her bid* on Ring)
  and drop from 74 to 29 points
  Bob would gain 40 (*his bid* on Ring)
  and go from 40 to 80 points.
  Alice gets the Ring back.

Adjusted Winner Procedure Step 6

6) If the Giver still has more points than the other person but cannot give any items, the two people must **share the item.**

	Alice	Bob	Point Ratio
Ring	**45**	40	1.125
Desk	21→	**20**	1.05
TV	5	**20**	4
Sofa	**29**	20	1.45
Points from Items Won	**74**	**40**	

- Alice and Bob must share the Ring, the item she tried to give to him.

	Alice	Bob	Point Ratio
Ring	**45**	40	1.125
Desk	21→	**20**	1.05
TV	5	**20**	4
Sofa	**29**	20	1.45
Points from Items Won	**74**	**40**	

- Alice and Bob must share the Ring, the item she tried to give to him.
- To **share an item** means that each person will **have a fraction (between 0 to 1)** of the item.
- The sum of the amount owned by both people must add up to 1.

- Alice and Bob must share the Ring, the item she tried to give to him. We determine how to split the item using a transfer equation.
- To **share an item** means that each person will **have a fraction (between 0 to 1)** of the item.
- The sum of the amount owned by both people must add up to 1.
- Each person gets back that fraction of their bid on the shared item.
- Goal: Split item so that both people have same amount of points.

Transfer Equation

- The sum of the amount owned by both people must add up to 1.
- Each person gets back that fraction of their bid on the shared item.
- Goal: Split item so that both people have same amount of points.

Let s be the fraction of a shared item owned by the Giver. We find s by solving the **transfer equation:**

$$\begin{aligned} & s \times (\text{Giver's bid on Shared item}) \\ & + (\text{Giver's points minus bid on the Shared item}) \\ = & (1-s) \times (\text{Other Person's Bid on Shared Item}) \\ & + (\text{Other person's points}) \end{aligned}$$

Transfer Equation

- Goal: Split item so that both people have same amount of points.

Let s be the fraction of a shared item owned by the Giver. We find s by solving the **transfer equation:**

$$\begin{aligned} & s \times (\text{Giver's bid on Shared item}) \\ & + (\text{Giver's points minus bid on the Shared item}) \\ = & (1-s) \times (\text{Other Person's Bid on Shared Item}) \\ & + (\text{Other person's points}) \end{aligned}$$

@Home Transfer Equation Details

- $s \times$ (Giver's Bid on Shared Item)
  - Giver gets back this fraction of bid on shared item.
- (Giver's points minus bid on the Shared item)
  - This is how many points the Giver has. When sharing the item, we take it from the giver for a moment.
- $(1 - s) \times$ (Other Person's Bid on Shared Item)
  - Other person gets remaining $1 - s$ fraction of shared item, and gets back this fraction of bid.
- (Other person's points)
  - This is how many points the other person has has so far.
- Remember, we want both people to have has the same amount of points, so we share an item to accomplish this.

?(5.3) Adjusted Winner Procedure Step 7

7) Write the transfer equation. The Ring is the Shared Item.

	Alice	Bob	Point Ratio
Ring	**45**	40	1.125
Desk	21→	**20**	1.05
TV	5	**20**	4
Sofa	**29**	20	1.45
Points from Items Won	**74**	**40**	

A) $s \times (45) + 74 = (1 - s) \times (40) + 40$
B) $s \times (45) + 29 = (1 - s) \times (40) + 40$
C) $s \times (45) + 74 = (1 - s) \times (20) + 40$
D) $s \times (45) + 29 = (1 - s) \times (40) + 0$
E) $s \times (45) + 29 = (1 - s) \times (20) + 40$

Adjusted Winner Procedure Step 7 Answer

7) Write the transfer equation.

$$
\begin{aligned}
& s \times (\text{Alice's bid on Ring}) \\
& + (\text{Alice's points from Items Won Except the Ring}) \\
= & (1-s) \times (\text{Bob's bid on Ring}) \\
& + (\text{Bob's points from Items Won})
\end{aligned}
$$

$$
\begin{aligned}
s \times (45) + (74-45) \\
= (1-s) \times (40) + 40
\end{aligned}
$$

7) Write the transfer equation.

$$
\begin{aligned}
& s \times (\text{Alice's bid on Ring}) \\
& + (\text{Alice's points from Items Won Except the Ring}) \\
=& (1-s) \times (\text{Bob's bid on Ring}) \\
& + (\text{Bob's points from Items Won})
\end{aligned}
$$

$$
\begin{aligned}
& s \times (45) + (74-45) \\
=& (1-s) \times (40) + 40
\end{aligned}
$$

$$
\begin{aligned}
& s \times (45) + (29) \\
=& (1-s) \times (40) + 40
\end{aligned}
$$

$$
\begin{aligned}
& s \times (\text{Alice's bid on Ring}) \\
& + (\text{Alice's points from Items Won Except the Ring}) \\
= & (1-s) \times (\text{Bob's bid on Ring}) \\
& + (\text{Bob's points from Items Won})
\end{aligned}
$$

$$
\begin{aligned}
& s \times (45) + (74-45) \\
= & (1-s) \times (40) + 40
\end{aligned}
$$

$$
\begin{aligned}
& s \times (45) + (29) \\
= & (1-s) \times (40) + 40
\end{aligned}
$$

Distribute on Bob's side:

$$45s + 29 = (1-s)(40) + 40$$

$$
\begin{aligned}
& s \times (\text{Alice's bid on Ring}) \\
& + (\text{Alice's points from Items Won Except the Ring}) \\
= & (1-s) \times (\text{Bob's bid on Ring}) \\
& + (\text{Bob's points from Items Won})
\end{aligned}
$$

$$
\begin{aligned}
& s \times (45) + (74-45) \\
= & (1-s) \times (40) + 40
\end{aligned}
$$

$$
\begin{aligned}
& s \times (45) + (29) \\
= & (1-s) \times (40) + 40
\end{aligned}
$$

Distribute on Bob's side:

$$
\begin{aligned}
45s + 29 &= (1-s)(40) + 40 \\
&= (1)(40) - (s)(40) + 40
\end{aligned}
$$

$$
\begin{aligned}
& s \times (\text{Alice's bid on Ring}) \\
& + (\text{Alice's points from Items Won Except the Ring}) \\
=& (1-s) \times (\text{Bob's bid on Ring}) \\
& + (\text{Bob's points from Items Won})
\end{aligned}
$$

$$
\begin{aligned}
& s \times (45) + (74-45) \\
=& (1-s) \times (40) + 40
\end{aligned}
$$

$$
\begin{aligned}
& s \times (45) + (29) \\
=& (1-s) \times (40) + 40
\end{aligned}
$$

Distribute on Bob's side:

$$
\begin{aligned}
45s + 29 &= (1-s)(40) + 40 \\
&= (1)(40) - (s)(40) + 40 \\
&= 40 - 40s + 40
\end{aligned}
$$

$$
\begin{aligned}
& s \times (\text{Alice's bid on Ring}) \\
& + (\text{Alice's points from Items Won Except the Ring}) \\
=& (1-s) \times (\text{Bob's bid on Ring}) \\
& + (\text{Bob's points from Items Won})
\end{aligned}
$$

$$
\begin{aligned}
& s \times (45) + (74-45) \\
=& (1-s) \times (40) + 40
\end{aligned}
$$

$$
\begin{aligned}
& s \times (45) + (29) \\
=& (1-s) \times (40) + 40
\end{aligned}
$$

Distribute on Bob's side:

$$
\begin{aligned}
45s + 29 &= (1-s)(40) + 40 \\
&= (1)(40) - (s)(40) + 40 \\
&= 40 - 40s + 40 \\
&= -40s + 80,
\end{aligned}
$$

$$s \times (45) + (74 - 45)$$
$$= (1 - s) \times (40) + 40$$

$$s \times (45) + (29)$$
$$= (1 - s) \times (40) + 40$$

Distribute on Bob's side:

$$\begin{aligned} 45s + 29 &= (1 - s)(40) + 40 \\ &= (1)(40) - (s)(40) + 40 \\ &= 40 - 40s + 40 \\ &= -40s + 80, \\ 45s + 29 &= -40s + 80 \end{aligned}$$

Adjusted Winner Procedure Step 8

8) Solve the transfer equation.

$$\begin{aligned} 45s+29 &= -40s+80 \\ +40s & \quad +40s \\ 85s+29 &= 80 \end{aligned}$$

Adjusted Winner Procedure Step 8

8) Solve the transfer equation.

$$\begin{array}{rcl} 45s+29 & = & -40s+80 \\ +40s & & +40s \\ 85s+29 & = & 80 \\ -29 & & -29 \\ 85s & = & 51 \end{array}$$

Adjusted Winner Procedure Step 8

8) Solve the transfer equation.

$$\begin{aligned} 45s+29 &= -40s+80 \\ +40s & \quad +40s \\ 85s+29 &= 80 \\ -29 & \quad -29 \\ 85s &= 51 \\ s &= \frac{51}{85} = 0.6 \end{aligned}$$

- The Giver, Alice, gets 0.6 (3/5ths) of the Ring.

Next time

- Fair (with a Capital F) Compensation Scenarios