Intro to Contemporary Math

New Method: Borda Count

Nicholas Nguyen
nicholas.nguyen@uky.edu

Department of Mathematics
UK

Agenda

- WebWork Homework Assignments
- Counting Ballots with Preference Schedules
- New Voting Method: The Borda Count
- Plurality Method Versus Borda Count

WebWork

Homework site:
`https://webwork.as.uky.edu/webwork2/MA111-F18-Nguyen/`

- Login Info:

  Log in through the link on our Canvas page.
  Check tonight for your first assignment.

Preference Ballots

Definition

A *preference ballot* is a ballot in which the voter ranks candidates/choices in order of preference.

Example

Between the colors Blue, Green, and Red, my most favorite is Blue and my least favorite is Red. So my ballot would look like

Nguyen
B
G
R

Multiple Preference Ballots

We combine multiple preference ballots into a *preference schedule*. The number at the top of each column shows how many people ranked their choices in that order.

4	2	3
B	G	B
G	B	R
R	R	G

?(2.1) Reading a Preference Schedule

4	2	3
B	G	B
G	B	R
R	R	G

How many people voted “Blue, Red, Green?”

Reading a Preference Schedule

4	2	3
B	G	B
G	B	R
R	R	G

- 3 people voted "Blue, Red, Green." Their votes are recorded in the third column.

?(2.2) Reading a Preference Schedule

4	2	3
B	G	B
G	B	R
R	R	G

How many second place votes did Green get?

Reading a Preference Schedule

4	2	3
B	G	B
G	B	R
R	R	G

- 4 people voted Green as their second place choice. Their votes are recorded in the first column.

?(2.3) Reading a Preference Schedule

4	2	3
B	G	B
G	B	R
R	R	G

How many first place votes did Blue get?

Reading a Preference Schedule

4	2	3
B	G	B
G	B	R
R	R	G

- 7 people voted Blue as their first place choice: 4 voters in the first column and 3 voters in the third column.

?(2.4) Creating a Preference Schedule

Here are four people's preference ballots.

Alice	Bob	Carol	Dan
B	G	B	B
G	B	R	G
R	R	G	R

Combine the ballots into a preference schedule. What numbers go above each column? Type the numbers from left to right.

?	?	?	?	?	?
G	G	R	R	B	B
B	R	G	B	G	R
R	B	B	G	R	G

Creating a Preference Schedule

Alice	Bob	Carol	Dan
B	G	B	B
G	B	R	G
R	R	G	R

Preference schedule:

1	0	0	0	2	1
G	G	R	R	B	B
B	R	G	B	G	R
R	B	B	G	R	G

Plurality Method with Preference Ballots

2	3	4
A	B	C
B	C	B
C	A	A

- The choice with the **most first place votes wins** if we **use the plurality method**. Here, Cobalt wins with 4 first place votes.

?(2.5) Who Won?

3	4	5	3
A	B	C	A
B	C	B	C
C	A	A	B

Select your answer by pressing the first letter of the color you think won on the Clicker.

Who Won?

3	4	5	**3**
A	B	C	**A**
B	C	B	C
C	A	A	B

Azure won with a total of 6 first place votes: 3 from the people in the first column, and 3 from the people in the fourth column.

New Method: Borda Count

Each voter ranks the candidates/choices. Say there are N candidates.
Each first place vote a candidate receives is worth N points.
Each second place vote a candidate receives is worth $N-1$ points.
Each third place vote a candidate receives is worth $N-2$ points, and so on.
Each last place vote is worth 1 point.
The candidate with the most points wins the election.

Borda Count Example A

2	3	4	
A	B	C	**3 points per vote**
B	C	B	**2 points per vote**
C	A	A	**1 point per vote**

- Azure gets 3 points per 1st place vote, 2 points per 2nd place vote, and 1 point per 3rd/last place vote.
- Thus, Azure gets

$(3+3)$

2	**3**	4	
A	B	C	**3 points per vote**
B	C	B	**2 points per vote**
C	**A**	A	**1 point per vote**

- Azure gets 3 points per 1st place vote, 2 points per 2nd place vote, and 1 point per 3rd/last place vote.
- Thus, Azure gets

$$(3+3)+(1+1+1)$$

2	3	**4**
A	B	C
B	C	B
C	A	**A**

3 points per vote
2 points per vote
1 point per vote

- Azure gets 3 points per 1st place vote, 2 points per 2nd place vote, and 1 point per 3rd/last place vote.
- Thus, Azure gets

$$(3+3)+(1+1+1)+(1+1+1+1)$$

2	3	4
A	B	C
B	C	B
C	A	A

3 points per vote
2 points per vote
1 point per vote

- Azure gets 3 points per 1st place vote, 2 points per 2nd place vote, and 1 point per 3rd/last place vote.
- Thus, Azure gets

$$\begin{aligned}(3+3)+(1+1+1)+(1+1+1+1) &= 2(3)+3(1)+4(1)\\ &= 13 \text{ points}\end{aligned}$$

@Home: Example with Four Candidates

2	3	4	2	5	
Azure	Blue	Cobalt	Cobalt	Denim	4 points
Denim	Cobalt	Blue	Denim	Blue	3 points
Cobalt	Azure	Azure	Blue	Azure	2 points
Blue	Denim	Denim	Azure	Cobalt	1 point

- How many Borda points did each color get? Which color wins with the Borda count method? Which color wins with the plurality method?

@Home: Example with Four Candidates

2	3	4	2	5	
Azure	Blue	Cobalt	Cobalt	Denim	4 points
Denim	Cobalt	Blue	Denim	Blue	3 points
Cobalt	Azure	Azure	Blue	Azure	2 points
Blue	Denim	Denim	Azure	Cobalt	1 point

Azure: $2(4)+3(2)+4(2)+2(1)+5(2)=34$ points

@Home: Example with Four Candidates

2	3	4	2	5	
Azure	Blue	Cobalt	Cobalt	Denim	4 points
Denim	Cobalt	Blue	Denim	Blue	3 points
Cobalt	Azure	Azure	Blue	Azure	2 points
Blue	Denim	Denim	Azure	Cobalt	1 point

Blue: $2(1) + 3(4) + 4(3) + 2(2) + 5(3) = 45$ points

@Home: Example with Four Candidates

2	3	4	2	5	
Azure	Blue	Cobalt	Cobalt	Denim	4 points
Denim	Cobalt	Blue	Denim	Blue	3 points
Cobalt	Azure	Azure	Blue	Azure	2 points
Blue	Denim	Denim	Azure	Cobalt	1 point

Cobalt: $2(2) + 3(3) + 4(4) + 2(4) + 5(1) = 42$ points

@Home: Example with Four Candidates

2	3	4	2	5	
Azure	Blue	Cobalt	Cobalt	Denim	4 points
Denim	Cobalt	Blue	Denim	Blue	3 points
Cobalt	Azure	Azure	Blue	Azure	2 points
Blue	Denim	Denim	Azure	Cobalt	1 point

Denim: $2(3)+3(1)+4(1)+2(3)+5(4)=39$ points

@Home: Example with Four Candidates

2	3	4	2	5	
Azure	Blue	Cobalt	Cobalt	Denim	4 points
Denim	Cobalt	Blue	Denim	Blue	3 points
Cobalt	Azure	Azure	Blue	Azure	2 points
Blue	Denim	Denim	Azure	Cobalt	1 point

- Blue gets the most points, so it wins with the Borda count method.
- However, Cobalt gets the most first place votes (6 from the voters in the third and fourth columns), so it wins with the Plurality method.