Intro to Contemporary Math

New Method: Plurality with Elimination

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Agenda

- Continue Borda Count
- New Voting Method: Plurality with Elimination (PwE)
- Plurality Method Versus Borda Count Versus PwE

Announcements

- Please create your REEF account before Friday.
- Part 2 of Homework 1 is up.
- You will have a 20-25 minute mini-exam (quiz) next Wednesday in class. It will cover up to this week's material.

Borda Count Part B

2	3	4	
A	B	C	**3 points per vote**
B	C	B	**2 points per vote**
C	A	A	**1 point per vote**

- Blue gets 3 points per 1st place vote, 2 points per 2nd place vote, and 1 point per 3rd/last place vote.
- Thus, Blue gets:

Borda Count Part B

2	3	4	
A	B	C	**3 points per vote**
B	C	B	**2 points per vote**
C	A	A	**1 point per vote**

- Blue gets 3 points per 1st place vote, 2 points per 2nd place vote, and 1 point per 3rd/last place vote.
- Thus, Blue gets:

$$2(2)+$$

Borda Count Part B

2	3	4
A	B	C
B	C	B
C	A	A

3 points per vote
2 points per vote
1 point per vote

- Blue gets 3 points per 1st place vote, 2 points per 2nd place vote, and 1 point per 3rd/last place vote.
- Thus, Blue gets:

$$2(2)+3(3)+$$

Borda Count Part B

2	3	4
A	B	C
B	C	B
C	A	A

3 points per vote
2 points per vote
1 point per vote

- Blue gets 3 points per 1st place vote, 2 points per 2nd place vote, and 1 point per 3rd/last place vote.
- Thus, Blue gets:

$$2(2)+3(3)+4(2) = 4+9+8$$
$$= \boxed{21 \text{ points}}$$

?(3.1) Borda Count Part C

2	3	4	
A	B	C	**3 points per vote**
B	C	B	**2 points per vote**
C	A	A	**1 point per vote**

How many points does Cobalt get?

Borda Count Part C

2	3	4	
A	B	C	**3 points per vote**
B	C	B	**2 points per vote**
C	A	A	**1 point per vote**

- Cobalt gets:

$$2(1)+3(2)+4(3) = 2+6+12$$
$$= \boxed{20 \text{ points}}$$

Borda Count Common Error

2	3	4	
A	B	C	**3 points per vote**
B	C	B	**2 points per vote**
C	A	A	**1 point per vote**

- If you got 6 points for Cobalt (1 for 3rd place in Column 1, plus 2 for 2nd place in Column 2, plus 3 for 1st place in Column 3), you must remember that each column can represent more than one voter (look at the number at the top of the column).

Different Results?

2	3	4
A	B	C
B	C	B
C	A	A

$$\begin{aligned} \text{Azure: } 2(3)+3(1)+4(1) &= 13 \text{ points} \\ \text{Blue: } 2(2)+3(3)+4(2) &= 21 \text{ points} \\ \text{Cobalt: } 2(1)+3(2)+4(3) &= 20 \text{ points} \end{aligned}$$

What helped Blue win using the Borda count method?

Different Results Discussion Comments From Class

Summary:

- Second place votes are much more valuable in Borda count than in Plurality.
- Last place is devastating (only one point at a time). Candidates that want to win with Borda count should avoid last place votes.
- The method rewards a candidate being above an opponent by giving more points, even if neither of them are in first.

Different Results Discussion Comments From Class Blue's Victory

2	3	4
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

- Blue won because of its second place votes. In fact, of the 21 points it got, 12 of them come from second place votes (from Columns 1 and 3), so over half of its point total comes from the second place votes, and less than half from first place.
- Also, no one ranked Blue in last place (notice Blue does not appear in the last row of the schedule).

Different Results Discussion Comments From Class Azure's Loss

2	3	4
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

- Azure got 13 points and came in last. It got 7 points from last place votes (in Columns 2 and 3).
- Like with Blue, over half of Azure's points came from last place votes.

Different Results Discussion Comments From Class Cobalt's Loss to Blue

2	3	4	
Azure	Blue	Cobalt	**3 points per vote**
Blue	Cobalt	Blue	**2 points per vote**
Cobalt	Azure	Azure	**1 point per vote**

Despite Cobalt getting the most 1st place votes, it lost to Blue. Let us compute the point totals from right to left:
Blue:

$$4(2)$$

Total so far: 8

Cobalt (first place votes give it a lead):

$$4(3)$$

Total so far: 12

Different Results Discussion Comments From Class Cobalt's Loss to Blue

2	3	4	
Azure	Blue	Cobalt	**3 points per vote**
Blue	Cobalt	Blue	**2 points per vote**
Cobalt	Azure	Azure	**1 point per vote**

Despite Cobalt getting the most 1st place votes, it lost to Blue. Let us compute the point totals from right to left:
Blue (just one point behind):

$$4(2)+3(3)$$

Total so far: 17

Cobalt:

$$4(3)+3(2)$$

Total so far: 18

Different Results Discussion Comments From Class Cobalt's Loss to Blue

2	3	4	
Azure	Blue	Cobalt	**3 points per vote**
Blue	Cobalt	Blue	**2 points per vote**
Cobalt	Azure	Azure	**1 point per vote**

Despite Cobalt getting the most 1st place votes, it lost to Blue. Let us compute the point totals from right to left:
Blue (pulls ahead for the victory):

$$4(2)+3(3)+2(2)$$

Total so far: 21

Cobalt (last place votes cost it the win):

$$4(3)+3(2)+2(1)$$

Total so far: 20

Plurality with Elimination

Using the preference schedule:

Step 1: Check for a candidate with **over** 50% of the first place votes.
If there is one, that candidate is the **winner**.
If not, go to **Step 2**.

Step 2: **Eliminate** the candidate with the fewest first place votes. Remove them from the schedule.

Step 3: **Push** the other candidates up to fill in the blanks. Then **go back to Step 1.**

?(3.2) PwE Example 1

2	3	6
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

Which color is eliminated first?

A) Azure
B) Blue
C) Cobalt
D) No one

PwE Example 1

2	3	6
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

No one! Cobalt has 6 out of 11 first place votes, which is more than 50%, so Cobalt **automatically** wins.

$$6/11 > 0.5\ (50\%)$$

?(3.3) PwE Example 2 Step 1

2	3	4
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

Which color is eliminated first?

A) Azure
B) Blue
C) Cobalt
D) No one

PwE Example 2 Step 1

2	3	4
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

This time, no color gets over 50% of the first place votes. There are $2+3+4=9$ voters, so the minimum needed for over 50% is

$$9+1=10,$$

$$\frac{10 \text{ voters}}{2}=5, \text{ rounds up to } \boxed{5 \text{ voters}}$$

PwE Example 2 Step 2

2	3	4
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

Thus, we must eliminate the candidate with the fewest first place votes, Azure.

PwE Example 2 Step 2

Remove Azure from schedule:

2	3	4
	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt		

PwE Example 2 Step 3

Push up

2	3	4
↑	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	↑	↑

PwE Example 2 Step 3

2	3	4
Blue	Blue	Cobalt
Cobalt	Cobalt	Blue

?(3.4) PwE Example 2 Part 2, Step 1

2	3	4
Blue	Blue	Cobalt
Cobalt	Cobalt	Blue

Now we repeat Step 1. Has a color won yet?

A) No one wins yet

B) Blue wins now

C) Cobalt wins now

PwE Example 2 Part 2, Step 1

2	3	4
Blue	Blue	Cobalt
Cobalt	Cobalt	Blue

Blue has $2+3=5$ first place votes now. This gives Blue over 50% of the first place votes, so it is the winner with PwE.

Plurality with Elimination

Summary:

- If there is a candidate with over 50% of the first place votes, that candidate wins.
- If not, then eliminate the candidate with the fewest first place votes and move the other candidates' rankings up to fill in spaces.
- Repeat until there is a candidate with over 50% of the first place votes

Alternate Names:

- Instant Runoff Voting (IRV)
- Hare Method (on WebWork)

Next Time

- Remember: Two homework assignments!
- PwE, and a Fourth voting method