Intro to Contemporary Math New Method: Plurality with Elimination

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Agenda

- Continue Borda Count
- ▶ New Voting Method: Plurality with Elimination (PwE)
- Plurality Method Versus Borda Count Versus PwE

Announcements

- Please create your REEF account before Friday.
- Part 2 of Homework 1 is up.
- You will have a 20-25 minute mini-exam (quiz) next Wednesday in class. It will cover up to this week's material.

2	3	4	
Α	В	С	3 points per vote
В	С	В	2 points per vote
С	Α	Α	1 point per vote

- ▶ Blue gets 3 points per 1st place vote, 2 points per 2nd place vote, and 1 point per 3rd/last place vote.
- ► Thus, Blue gets:

2	3	4	
Α	В	С	3 points per vote
B	С	В	2 points per vote
С	Α	Α	1 point per vote

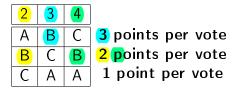
- ▶ Blue gets 3 points per 1st place vote, 2 points per 2nd place vote, and 1 point per 3rd/last place vote.
- ► Thus, Blue gets:

$$(2(2) +$$

2	3	4	
Α	В	С	3 points per vote
B	С	В	2 points per vote
С	Α	Α	1 point per vote

- ▶ Blue gets 3 points per 1st place vote, 2 points per 2nd place vote, and 1 point per 3rd/last place vote.
- ▶ Thus, Blue gets:

$$2(2) + 3(3) +$$



- ▶ Blue gets 3 points per 1st place vote, 2 points per 2nd place vote, and 1 point per 3rd/last place vote.
- ► Thus, Blue gets:

$$2(2)+3(3)+4(2) = 4+9+8$$

= 21 points

?(3.1) Borda Count Part C

2	3	4	
Α	В	С	3 points per vote
В	C	В	2 points per vote
С	Α	Α	1 point per vote

How many points does Cobalt get?

2	3	4	
Α	В	C	3 points per vote
В	C	В	2 points per vote
C	Α	Α	1 point per vote

► Cobalt gets:

$$2(1)+3(2)+4(3) = 2+6+12$$

= 20 points

Borda Count Common Error

2	3	4	
Α	В	С	3 points per vote
В	С	В	2 points per vote
С	Α	Α	1 point per vote

▶ If you got 6 points for Cobalt (1 for 3rd place in Column 1, plus 2 for 2nd place in Column 2, plus 3 for 1st place in Column 3), you must remember that each column can represent more than one voter (look at the number at the top of the column).

Different Results?

2	3	4
Α	В	С
В	С	В
С	Α	Α

Azure: 2(3) + 3(1) + 4(1) = 13 points

Blue: 2(2) + 3(3) + 4(2) = 21 points

Cobalt: 2(1) + 3(2) + 4(3) = 20 points

What helped Blue win using the Borda count method?

Different Results Discussion Comments From Class

Summary:

- Second place votes are much more valuable in Borda count than in Plurality.
- Last place is devastating (only one point at a time). Candidates that want to win with Borda count should avoid last place votes.
- ► The method rewards a candidate being above an opponent by giving more points, even if neither of them are in first.

Different Results Discussion Comments From Class Blue's Victory

2	3	4
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

- ▶ Blue won because of its second place votes. In fact, of the 21 points it got, 12 of them come from second place votes (from Columns 1 and 3), so over half of its point total comes from the second place votes, and less than half from first place.
- ▶ Also, no one ranked Blue in last place (notice Blue does not appear in the last row of the schedule).

Different Results Discussion Comments From Class Azure's Loss

2	3	4
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

- ► Azure got 13 points and came in last. It got 7 points from last place votes (in Columns 2 and 3).
- ► Like with Blue, over half of Azure's points came from last place votes.

Different Results Discussion Comments From Class Cobalt's Loss to Blue

2	3	4
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

3 points per vote 2 points per vote 1 point per vote

Despite Cobalt getting the most 1st place votes, it lost to Blue. Let us compute the point totals from right to left: Blue:

4(2)

Total so far: 8

Cobalt (first place votes give it a lead):

4(3)

Total so far: 12



Different Results Discussion Comments From Class Cobalt's Loss to Blue

2	3	4
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

3 points per vote 2 points per vote 1 point per vote

Despite Cobalt getting the most 1st place votes, it lost to Blue. Let us compute the point totals from right to left: Blue (just one point behind):

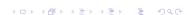
$$4(2) + 3(3)$$

Total so far: 17

Cohalt:

$$4(3) + 3(2)$$

Total so far: 18



Different Results Discussion Comments From Class Cobalt's Loss to Blue

2	3	4
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

3 points per vote 2 points per vote 1 point per vote

Despite Cobalt getting the most 1st place votes, it lost to Blue. Let us compute the point totals from right to left: Blue (pulls ahead for the victory):

$$4(2) + 3(3) + 2(2)$$

Total so far: 21

Cobalt (last place votes cost it the win):

$$4(3) + 3(2) + 2(1)$$

Total so far: 20



Plurality with Elimination

Using the preference schedule:

- Step 1: Check for a candidate with **over** 50% of the first place votes.

 If there is one, that candidate is the **winner**.

 If not, go to **Step 2**.
- Step 2: Eliminate the candidate with the fewest first place votes. Remove them from the schedule.
- Step 3: Push the other candidates up to fill in the blanks. Then go back to Step 1.

?(3.2) PwE Example 1

2	3	6
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

Which color is eliminated first?

- A) Azure
- B) Blue
- C) Cobalt
- D) No one

PwE Example 1

2	3	6
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

No one! Cobalt has 6 out of 11 first place votes, which is more than 50%, so Cobalt **automatically** wins.

$$6/11 > 0.5 (50\%)$$

?(3.3) PwE Example 2 Step 1

2	3	4
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

Which color is eliminated first?

- A) Azure
- B) Blue
- C) Cobalt
- D) No one

2	3	4
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

This time, no color gets over 50% of the first place votes. There are 2+3+4=9 voters, so the minimum needed for over 50% is

$$9+1=10,$$
 $\frac{10 \text{ voters}}{2}=5, \text{ rounds up to } \boxed{5 \text{ voters}}$

2	3	4
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

Thus, we must eliminate the candidate with the fewest first place votes, Azure.

Remove Azure from schedule:

2	3	4
	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt		

Push up

2	3	4
\uparrow	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	\uparrow	†

2	3	4
Blue	Blue	Cobalt
Cobalt	Cobalt	Blue

?(3.4) PwE Example 2 Part 2, Step 1

2	3	4
Blue	Blue	Cobalt
Cobalt	Cobalt	Blue

Now we repeat Step 1. Has a color won yet?

- A) No one wins yet
- B) Blue wins now
- C) Cobalt wins now

PwE Example 2 Part 2, Step 1

2	3	4
Blue	Blue	Cobalt
Cobalt	Cobalt	Blue

Blue has 2+3=5 first place votes now. This gives Blue over 50% of the first place votes, so it is the winner with PwE.

Plurality with Elimination

Summary:

- ▶ If there is a candidate with over 50% of the first place votes, that candidate wins.
- ▶ If not, then eliminate the candidate with the fewest first place votes and move the other candidates' rankings up to fill in spaces.
- Repeat until there is a candidate with over 50% of the first place votes

Alternate Names:

- Instant Runoff Voting (IRV)
- Hare Method (on WebWork)

Next Time

- ▶ Remember: Two homework assignments!
- PwE, and a Fourth voting method