Intro to Contemporary Math

New Method: Pairwise Comparison

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Agenda

- PwE Discussion
- Pairwise Comparison

Announcements

- Your first mini-exam is next Wednesday. It covers the four voting methods.

Homework due Friday and next Monday!

?(4.1) PwE Example 2 Discussion

Beginning:

2	3	4
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

After Azure is eliminated:

2	3	4
Blue	Blue	Cobalt
Cobalt	Cobalt	Blue

Blue won the election, but at the beginning, it did not have enough 1st place votes to win. From which column did Blue get **additional** 1st place votes **after Azure's elimination**?

A) First column
B) Second column
C) Third column

PwE Example 2 Discussion

Beginning:

2	3	4
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

After Azure is eliminated:

2	3	4
Blue	Blue	Cobalt
Cobalt	Cobalt	Blue

Column 1:
Originally, Blue got second place votes from Column 1, but after Azure's elimination, Blue got pushed into 1st place in that column (taking Azure's place). In a way, **Blue got 2 more 1st place votes** that used to belong to Azure.

PwE Example 2 Discussion

Beginning:

2	3	4
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

After Azure is eliminated:

2	3	4
Blue	Blue	Cobalt
Cobalt	Cobalt	Blue

Blue also got 2nd place votes from Column 3, but since Cobalt never got eliminated, those 2nd place votes **never** became 1st place votes for Blue.

PwE Example 2 Discussion

Getting 1st place votes still matters - otherwise the candidate could risk elimination!

?(4.2) Possible 1st Place Vote?

Ballot
D
A
C
B

Is it possible for enough candidates to be eliminated so that this ballot gives a 1st place vote to Blue?
Type "yes" or "no" please.
Hint: Can PwE eliminate all but one candidate ever? What happens when there are two left? Can they both have less than 50% of the 1st place votes?

Possible 1st Place Vote?

No. At worst, PwE can go down to two candidates left. At this point, unless there is a tie, one of them must have over 50% of the 1st place votes.
This means that in PwE, last place votes will never rise up to 1st place votes.

PwE Important Points

- **Last place votes are useless**. PwE cannot eliminate enough candidates for someone to rise from last to 1st in a column.
- Rankings below 1st and above last (especially 2nd place) are only helpful if they are below unpopular candidates.

PwE "Strategies"

A candidate who wants to win an election that uses PwE can either:

- Try to be a majority candidate at the beginning, or
- Get popular among supporters of unpopular opponents so that when they get eliminated, the candidate gets their 1st place votes.

Pairwise Comparison Method

- Look at every pair of candidates in a **Pairwise Comparison**. Each pairwise comparison is like a mini one-on-one election between two candidates.
- We assign points based on how candidates perform in the comparisons.
- The candidate with the most points wins the election.

Pairwise Comparisons Between Candidates

We can compare two candidates A and B to see who is more popular between the two in a **Pairwise Comparison**.

- All the voters who rank A higher than B are in **A's camp**
- All the voters who rank B higher than A are in **B's camp**
- The candidate whose **camp has more voters wins**. If both camps have the **same number of voters**, the comparison ends in a **tie**.

?(4.3) Camps

Here are five ballots from an election with 3 candidates. For the A vs. B comparison, each voter will be placed in either A or B's camp depending on who the voter likes more.

Alice	Bob	Carol	Dan	Ed
A	C	B	A	B
B	A	C	B	A
C	B	A	C	C

Camp:

For each voter, determine which camp they are in: A's camp or B's camp. Type the letters A or B from left to right.

Camps

Alice	Bob	Carol	Dan	Ed
A	C	B	A	B
B	A	C	B	A
C	B	A	C	C

Camp: A A B A B

- Alice, Bob, and Dan go in A's camp. These 3 voters listed A above B on their ballots.
- Carol and Ed go in B's camp. These 2 voters listed B above A on their ballots.

Pairwise Comparison Using Pref. Schedules

2	4	4	3
Azure	Blue	Cobalt	Azure
Cobalt	Azure	Blue	Blue
Blue	Cobalt	Azure	Cobalt

Let us start with Azure vs. Blue.

- Voters in Azure's camp rank Azure **above** Blue on their **ballots**.
  These 5 voters are in the **first** and **fourth** columns.
- 8 voters in Blue's camp (**second**, **third** columns)
- Blue wins this comparison

Pairwise Comparison Using Pref. Schedules

2	4	4	3
Azure	Blue	Cobalt	Azure
Cobalt	Azure	Blue	Blue
Blue	Cobalt	Azure	Cobalt

Azure vs. Cobalt:

- 9 voters in Azure's camp (**first**, **second**, and **fourth** columns)
- 4 voters in Cobalt's camp (**third** column)
- Azure wins this comparison

?(4.4) Pairwise Comparison Using Pref. Schedules

2	4	4	3
Azure	Blue	Cobalt	Azure
Cobalt	Azure	Blue	Blue
Blue	Cobalt	Azure	Cobalt

Which color wins the Blue vs. Cobalt comparison? Type its name or type "Tie."

Pairwise Comparison Using Pref. Schedules

2	4	4	3
Azure	Blue	Cobalt	Azure
Cobalt	Azure	Blue	Blue
Blue	Cobalt	Azure	Cobalt

Blue vs. Cobalt:

- 7 voters in Blue's camp (**second**, **fourth** columns)
- 6 voters in Cobalt's camp (**first**, **third** columns)
- Blue wins this comparison

Pairwise Comparison Using Pref. Schedules

2	4	4	3
Azure	Blue	Cobalt	Azure
Cobalt	Azure	Blue	Blue
Blue	Cobalt	Azure	Cobalt

If you got 2 in Blue's camp and 2 in Cobalt's camp, you must remember that each column can represent more than one voter.

Next Time

- Remember: Homework due Friday and next Monday
- Fourth voting method