Intro to Contemporary Math

Pairwise Comparison and Mini-Review

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Agenda

- Pairwise Comparison
- Mini-Review

Announcements

- You will have a 20-25 minute mini-exam (quiz) on Wednesday in class.

Look out for an announcement about the mini-exam on Tuesday.

Homework 1 Part 1 due tonight!
Homework 1 Part 2 due Monday.

Reminder: check in on REEF at the beginning of class.

Pairwise Comparisons Between Candidates

We can compare two candidates A and B to see who is more popular between the two in a **Pairwise Comparison**.

- All the voters who rank A higher than B are in **A's camp**
- All the voters who rank B higher than A are in **B's camp**
- The candidate whose **camp has more voters wins**. If both camps have the **same number of voters**, the comparison ends in a **tie**.

Pairwise Comparison Method Summary

The method of Pairwise Comparison works as follows:

- We look at every pair of candidates in a **Pairwise Comparison**.
- In each comparison, the candidate with the larger camp wins.
- Each comparison gives one point to its winner. Tied comparisons give half a point to both candidates.
- The candidate with the most points from winning (and tying) comparisons wins the election.
- Condorcet candidates win all of their comparisons. They not only got the most points, they crushed their competition.

?(4.4) Pairwise Comparison Using Pref. Schedules

2	4	4	3
Azure	Blue	Cobalt	Azure
Cobalt	Azure	Blue	Blue
Blue	Cobalt	Azure	Cobalt

Which color wins the Blue vs. Cobalt comparison? Type its name or type "Tie."

Pairwise Comparison Using Pref. Schedules

2	4	4	3
Azure	Blue	Cobalt	Azure
Cobalt	Azure	Blue	Blue
Blue	Cobalt	Azure	Cobalt

Blue vs. Cobalt:

- 7 voters in Blue's camp (**second**, **fourth** columns)
- 6 voters in Cobalt's camp (**first**, **third** columns)
- Blue wins this comparison

Pairwise Comparison Using Pref. Schedules

2	4	4	3
Azure	Blue	Cobalt	Azure
Cobalt	Azure	Blue	Blue
Blue	Cobalt	Azure	Cobalt

If you got 2 in Blue's camp and 2 in Cobalt's camp, you must remember that each column can represent more than one voter.

Pairwise Comparison Results

Pairs	Camps	Winner
A vs. B	5 to 8	Blue
A vs. C	9 to 4	Azure
B vs. C	7 to 6	Blue

- Each comparison gives one point to the winner.
- Tied comparisons give half a point to both candidates.

A gets 1 points, B gets 2 points, C gets 0 points.

Blue wins with the method of Pairwise Comparison.

Condorcet Candidate

If Candidate A always beats its opponent in the comparisons **A is involved in**, then Candidate A is a **Condorcet candidate (winner).**

Fact

In an election with N candidates, a Condorcet candidate must get $N-1$ points from winning comparisons against its $N-1$ opponents.

?(5.1) Is It a Condorcet Candidate?

Is Blue a Condorcet candidate? Type “yes” or “no” please.

Is It a Condorcet Candidate?

Yes. Blue won all the comparisons **it was involved in.**
It got $3-1=2$ points in an election with 3 candidates.
This makes Blue not only the winner with Pairwise comparison, but also a Condorcet candidate.

?(5.2) Pairwise Example

2	1	1
Azure	Blue	Cobalt
Cobalt	Azure	Blue
Blue	Cobalt	Azure

Pairs	Camps	Winner
A vs. B		
A vs. C		
B vs. C		

Who wins the A vs. B comparison? Type its name, or type "tie."

Pairwise Example

2	1	1
Azure	Blue	Cobalt
Cobalt	Azure	Blue
Blue	Cobalt	Azure

Pairs	Camps	Winner
A vs. B	2 to 2	Tie
A vs. C		
B vs. C		

A vs. B:

2 voters in A's camp (Column 1)

2 voters in B's camp (Columns 2 and 3)

Result: Tie

Pairwise Example

(2)	1	1
Azure	Blue	Cobalt
Cobalt	Azure	Blue
Blue	Cobalt	Azure

Pairs	Camps	Winner
A vs. B	2 to 2	Tie
A vs. C		
B vs. C		

Note that the first column has two voters. This is why A vs. B ends in a tie, not a win for Blue.

?(5.3) Pairwise Example

2	1	1
Azure	Blue	Cobalt
Cobalt	Azure	Blue
Blue	Cobalt	Azure

Pairs	Camps	Winner
A vs. B	2 to 2	Tie
A vs. C		
B vs. C		

Who wins the A vs. C comparison?
Type its name, or type "tie."

Pairwise Example

2	1	1
Azure	Blue	Cobalt
Cobalt	Azure	Blue
Blue	Cobalt	Azure

Pairs	Camps	Winner
A vs. B	2 to 2	Tie
A vs. C	3 to 1	Azure
B vs. C		

A vs. C:

3 voters in A's camp (Columns 1 and 2)

1 voter in C's camp (Column 3)

Result: A wins

?(5.4) Pairwise Example

2	1	1
Azure	Blue	Cobalt
Cobalt	Azure	Blue
Blue	Cobalt	Azure

Pairs	Camps	Winner
A vs. B	2 to 2	Tie
A vs. C	3 to 1	Azure
B vs. C		

Who wins the B vs. C comparison?
Type its name, or type "tie."

Pairwise Example

2	1	1
Azure	Blue	Cobalt
Cobalt	Azure	Blue
Blue	Cobalt	Azure

Pairs	Camps	Winner
A vs. B	2 to 2	Tie
A vs. C	3 to 1	Azure
B vs. C	1 to 3	Cobalt

B vs. C:

1 voter in B's camp (Column 2)

3 voters in C's camp (Columns 1 and 3)

Result: C wins

?(5.5) Pairwise Example

2	1	1
Azure	Blue	Cobalt
Cobalt	Azure	Blue
Blue	Cobalt	Azure

Pairs	Camps	Winner
A vs. B	2 to 2	Tie
A vs. C	3 to 1	Azure
B vs. C	1 to 3	Cobalt

How many points does Azure get? Type a number, using a decimal point if needed.

$$Half = \frac{1}{2} = 0.5$$

Example of Ties, No Condorcet Candidate

2	1	1
Azure	Blue	Cobalt
Cobalt	Azure	Blue
Blue	Cobalt	Azure

Pairs	Camps	Winner
A vs. B	2 to 2	Tie
A vs. C	3 to 1	Azure
B vs. C	1 to 3	Cobalt

- Azure gets 1.5 points for beating Cobalt and tying with Blue
- Blue gets 0.5 points for tying with Azure
- Cobalt gets 1 point for beating Blue
- Azure **wins** with Pairwise comparison, but is **not** a Condorcet candidate (only got 1.5 points, tied with Blue)

Winners and Losers of Pairwise Comparison

If you are in an election, in any comparison involving you,

- Voters who give you **1st** place votes will **always be in your camps**. These voters rank you **above** all of your opponents.
- Voters who give you **last** place votes will **never be in your camps**. These voters rank you **below** all of your opponents. Last place votes are dangerous in this method!

Overall, to win comparisons, and to win an election using Pairwise Comparison, you want most of the voters to rank you above the opponents.

?(5.6) Ballots That Help Win Plurality

Ballot 1	Ballot 2	Ballot 3	Ballot 4	Ballot 5
A	C	A	D	B
B	A	D	C	D
C	D	B	B	A
D	B	C	A	C

Which ballot helps Candidate B win with the Plurality method? Press the ballot number.

Ballots That Help Win Plurality

Ballot 1	Ballot 2	Ballot 3	Ballot 4	Ballot 5
A	C	A	D	B
B	A	D	C	D
C	D	B	B	A
D	B	C	A	C

Ballot 5:
Only 1st place votes matter in Plurality, so the answer is the ballot that gives B a 1st place vote.

?(5.7) Ballots and Borda Points

Ballot 1	Ballot 2	Ballot 3	Ballot 4	Ballot 5
A	C	A	D	B
B	A	D	C	D
C	D	B	B	A
D	B	C	A	C

Which ballot gives Candidate C 3 Borda points? Watch out: there are four candidates. Press the ballot number.

Ballots and Borda Points

Ballot 1	Ballot 2	Ballot 3	Ballot 4	Ballot 5
A	C	A	D	B
B	A	D	C	D
C	D	B	B	A
D	B	C	A	C

Ballot 4:

Last place votes give 1 point each, and as you go up a ballot, you go by 1 point, so with 4 candidates, 2nd place votes give 3 points.

?(5.8) Ballots and Getting More 1st Place Votes

Ballot 1	Ballot 2	Ballot 3	Ballot 4	Ballot 5
A	C	A	D	B
B	A	D	C	D
C	D	B	B	A
D	B	C	A	C

Which ballot will give Candidate B a **new** 1st place vote after Candidates C and D are eliminated? Press the ballot number.

Ballots and Getting More 1st Place Votes

Ballot 1	Ballot 2	Ballot 3	Ballot 4	Ballot 5
A		A		B
B	A			
		B	B	A
	B		A	

Ballot 4:
When C and D are eliminated, B will also be at the top of Ballot 4 as well. This gives B a new 1st place vote from Ballot 4 that it did not have before the eliminations.

?(5.9) Ballots and Camps

Ballot 1	Ballot 2	Ballot 3	Ballot 4	Ballot 5
A	C	A	D	B
B	A	D	C	D
C	D	B	B	A
D	B	C	A	C

Which ballot comes from a voter who:

- Always goes into A's camp in the A vs. anyone else comparisons
- Goes into C's camp in the C vs. D comparison

Press the ballot number.

Ballots and Camps

Ballot 1	Ballot 2	Ballot 3	Ballot 4	Ballot 5
A	C	A	D	B
B	A	D	C	D
C	D	B	B	A
D	B	C	A	C

Ballot 1:

The voter:

- Must have ranked A at the very top of the ballot above everyone else
- Ranked C above D

Only Ballot 1 has A at the top and C above D.

Next Time

- Remember: Homework due tonight
- Mini-exam
- Fairness in voting