Intro to Contemporary Math

Conditional Probability and Tables

Department of Mathematics
UK

October 8, 2018

Announcement

Homework due tonight.

Conditional Probability Computation

Let E and F be events in a sample space Ω. Then $P(F|E)$, the probability of getting an outcome in event F given that the outcome is in event E, is

$$P(F|E) = \frac{\text{Number of outcomes in } E \bigcap F}{\text{Number of outcomes in } E}.$$

Hint: The given event appears in the denominator.
This formula is very useful when working with tables.

Independent Events

Let E and F be events in a sample space Ω. Then E and F are said to be **independent events** if

$$P(F|E) = P(F).$$

That is, the conditional probability of F given that E occurred is the same as the (regular) probability of F.
The probability of F occurring is the same whether or not E occurs.

Checking Independence

Let E and F be events in a sample space Ω.
To check if E and F are independent events,

1. Compute $P(E)$, $P(F)$, and $P(F|E)$.
2. Compare $P(F|E)$ and $P(F)$. Are they **equal** or **not**?

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Checking Independence

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To check if E and F are independent events,

1. Compute $P(E)$, $P(F)$, and $P(F|E)$.
2. Compare $P(F|E)$ and $P(F)$. Are they **equal** or **not**?

- If $P(F|E)$ and $P(F)$ are **equal**, then E and F **are independent** events.
- If $P(F|E)$ and $P(F)$ are **different** numbers, then E and F **are not independent** events.

Home and Away Games

The stats for a sports team's season are given in the table. It records wins and losses in home and away games:

	Wins	Losses	Total
Home	21	3	24
Away	11	5	16
Total	32	8	40

Home and Away Games (Counts)

The stats for a sports team's season are given in the table. It records wins and losses in home and away games:

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For example,

- The team played 21 games that were winning home games.
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- There were a total of 24 home games.
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- There were a total of 24 home games.
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Away	11	5	16
Total	32	8	40

For example,

- The team played 21 games that were winning home games.
- There were a total of 24 home games.
- The team won 32 games.
- The team played 40 games in total.

Home and Away Games (Events)

The stats for a sports team's season are given in the table. It records wins and losses in home and away games:

	Wins	Losses	Total
Home	21	3	24
Away	11	5	16
Total	32	8	40

Suppose a game is chosen at random.

- Let E be the event "the game was a home game."
- Let F be the event "the team won the game."

Home and Away Games (E)

The stats for a sports team's season are given in the table. It records wins and losses in home and away games:

	Wins	Losses	Total
Home	21	3	**24**
Away	11	5	16
Total	32	8	**40**

The probability that the team played at home is

$$P(E) = \frac{24 \text{ home games}}{40 \text{ games total}} = \frac{24}{40}.$$

Home and Away Games (F)

The stats for a sports team's season are given in the table. It records wins and losses in home and away games:

	Wins	Losses	Total
Home	21	3	24
Away	11	5	16
Total	**32**	8	**40**

The probability that the team won is

$$P(F) = \frac{\text{32 winning games}}{\text{40 games total}} = \frac{32}{40}.$$

Home and Away Games ($E \bigcap F$)

The stats for a sports team's season are given in the table. It records wins and losses in home and away games:

	Wins	Losses	Total
Home	**21**	3	24
Away	11	5	16
Total	32	8	**40**

The probability that the game took place at home **and** was a winning game is

$$P(E \bigcap F) = \frac{\text{21 games won at home}}{\text{40 games total}} = \frac{21}{40}.$$

Home and Away Games $(F|E)$

The stats for a sports team's season are given in the table. It records wins and losses in home and away games:

	Wins	Losses	Total
Home	**21**	3	**24**
Away	11	5	16
Total	32	8	40

The probability that the team won **given** that it was a home game is

$$P(F|E) = \frac{\text{21 games won at home}}{\text{24 home games}}$$

Home and Away Games $(F|E)$

The stats for a sports team's season are given in the table. It records wins and losses in home and away games:

	Wins	Losses	Total
Home	**21**	3	**24**
Away			
Total			

The probability that the team won **given** that it was a home game is

$$P(F|E) = \frac{\text{21 games won at home}}{\text{24 home games}} = \frac{21}{24}.$$

The given event tells us to restrict to the row of home games.

Home and Away Games $(E|F)$

The stats for a sports team's season are given in the table. It records wins and losses in home and away games:

	Wins	Losses	Total
Home	**21**	3	24
Away	11	5	16
Total	**32**	8	40

The probability that the game was at home **given** that the team won is

$$P(E|F) = \frac{\text{21 games won at home}}{\text{32 winning games}}$$

Home and Away Games $(E|F)$

The stats for a sports team's season are given in the table. It records wins and losses in home and away games:

	Wins	Losses	Total
Home	**21**		
Away	11		
Total	**32**		

The probability that the game was at home **given** that the team won is

$$P(E|F) = \frac{\text{21 games won at home}}{\text{32 winning games}} = \frac{21}{32}.$$

The given event tells us to restrict to the column of wins.

Home and Away Games: Independence

To check if the events E, "the game was a home game," and F, "the team won the game," are independent, we compare $P(F)$ and $P(F|E)$.

$$P(F) = \frac{32}{40} = 0.8; \text{ but } P(F|E) = \frac{21}{24} = 0.875.$$

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Since $P(F)$ and $P(F|E)$ are **not equal**, the events are not independent.

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$$P(F) = \frac{32}{40} = 0.8; \text{ but } P(F|E) = \frac{21}{24} = 0.875.$$

Since $P(F)$ and $P(F|E)$ are **not equal**, the events are not independent.
Notice that the team is more likely to win their home games: $P(F|E)$ is a little larger than $P(F)$.

Home and Away Games ($E \cup F$)

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Total	32	8	40

To find the probability that the game was at home **or** a winning game, we count:

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	Wins	Losses	Total
Home	**21**	3	24
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Total	32	8	40

To find the probability that the game was at home **or** a winning game, we count:
the 21 games which were winning home games,

Home and Away Games $(E \cup F)$

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	Wins	Losses	Total
Home	**21**	**3**	24
Away	11	5	16
Total	32	8	40

To find the probability that the game was at home **or** a winning game, we count:
the 21 games which were winning home games,
the 3 other games that took place at home, and

Home and Away Games ($E \cup F$)

The stats for a sports team's season are given in the table. It records wins and losses in home and away games:

	Wins	Losses	Total
Home	**21**	**3**	24
Away	**11**	5	16
Total	32	8	40

To find the probability that the game was at home **or** a winning game, we count:
the 21 games which were winning home games,
the 3 other games that took place at home, and
the 11 other winning games.

Home and Away Games ($E \cup F$)

The stats for a sports team's season are given in the table. It records wins and losses in home and away games:

	Wins	Losses	Total
Home	**21**	**3**	24
Away	**11**	5	16
Total	32	8	**40**

To find the probability that the game was at home **or** a winning game, we count:
the 21 games which were winning home games,
the 3 other games that took place at home, and
the 11 other winning games.
The union is an event in the sample space of all 40 games.

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Home	**21**	**3**	24
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To find the probability that the game was at home **or** a winning game, we count:
the 21 games which were winning home games,
the 3 other games that took place at home, and
the 11 other winning games.
The union is an event in the sample space of all 40 games.

$$P(E \bigcup F) = \frac{21+3+11}{40 \text{ games total}} = \frac{35}{40}.$$

The Star Player

The stats for a sports team's season are given in the table. This team has a star player who got injured in the middle of the season. The table records wins and losses and whether the star player was in or not in the game:

	Wins	Losses	Total
Star in	12	4	16
Star out	15	5	20
Total	27	9	36

The Star Player

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	Wins	Losses	Total
Star in	12	4	16
Star out	15	5	20
Total	27	9	36

Suppose a game is chosen at random.

- Let E be the event "the star player was in the game."
- Let F be the event "the team won the game."

?(7.1) The Star Player (E)

	Wins	Losses	Total
Star in	12	4	16
Star out	15	5	20
Total	27	9	36

What is the probability of picking a game that the star player played in?

The Star Player (E)

	Wins	Losses	Total
Star in	12	4	**16**
Star out	15	5	20
Total	27	9	**36**

The probability that the star player played is

$$P(E) = \frac{\text{16 games with star}}{\text{36 games total}} = \frac{16}{36}.$$

?(7.2) The Star Player (F)

	Wins	Losses	Total
Star in	12	4	16
Star out	15	5	20
Total	27	9	36

What is the probability of picking a winning game?

The Star Player (F)

	Wins	Losses	Total
Star in	12	4	16
Star out	15	5	20
Total	**27**	9	**36**

The probability that the team won is

$$P(F) = \frac{27 \text{ winning games}}{36 \text{ games total}} = \frac{27}{36}.$$

?(7.3) The Star Player ($B \bigcap F$)

	Wins	Losses	Total
Star in	12	4	16
Star out	15	5	20
Total	27	9	36

What is the probability of picking a game that has the star player in it **and** was a winning game?

The Star Player $(E \bigcap F)$

	Wins	Losses	Total
Star in	**12**	4	16
Star out	15	5	20
Total	27	9	**36**

The probability that the game had the star player **and** was a winning game is

$$P(E \bigcap F) = \frac{\text{12 winning games with star}}{\text{36 games total}} = \frac{12}{36}.$$

?(7.4) The Star Player (?|?)

	Wins	Losses	Total
Star in	12	4	16
Star out	15	5	20
Total	27	9	36

What is the probability of picking a winning game given that the chosen game has the star player in it?

The Star Player $(F|E)$

	Wins	Losses	Total
Star in	**12**	4	**16**
Star out			
Total			

The probability that the team won **given** that the star player was in is

$$P(F|E) = \frac{12 \text{ winning games with star}}{16 \text{ games with star}} = \frac{12}{16}.$$

?(7.5) The Star Player (?|?)

	Wins	Losses	Total
Star in	12	4	16
Star out	15	5	20
Total	27	9	36

What is the probability of picking a game that has the star player in it given that the chosen game was a winning game?

The Star Player $(E|F)$

	Wins	Losses	Total
Star in	**12**		
Star out	15		
Total	**27**		

The probability that the star player was in **given** that the team won is

$$P(E|F) = \frac{\text{12 winning games with star}}{\text{27 winning games}} = \frac{12}{27}.$$

?(7.6) The Star Player: Independence

- Let E be the event "the star player was in the game."
- Let F be the event "the team won the game."

Our computations so far:

- $P(E) = \frac{16}{36}$
- $P(F) = \frac{27}{36}$
- $P(F|E) = \frac{12}{16}$
- $P(E|F) = \frac{12}{27}$

Based on these calculations, are the events E and F independent?

The Star Player: Independence

Let E be the event "the star player was in the game."
Let F be the event "the team won the game."
The events are independent, because

$$P(F) = \frac{27}{36}, \text{ or } 0.75$$

is equal to

$$P(F|E) = \frac{12}{16}, \text{ or } 0.75.$$

It looks like the team's likelihood of winning over the season did not get affected by the star player's sidelining.

?(7.7) The Star Player ($E \cup F$)

	Wins	Losses	Total
Star in	12	4	16
Star out	15	5	20
Total	27	9	36

What is the probability of picking a game that has the star player in it or was a winning game?

The Star Player $(E \bigcup F)$

	Wins	Losses	Total
Star in	**12**	**4**	16
Star out	**15**	5	20
Total	27	9	**36**

To find the probability that the game had the star player, **or** was a winning game, we count:
the 12 games which were winning games with the star,
the 4 other games the star played in, and
the 15 other winning games:

$$P(E \bigcup F) = \frac{12 + 4 + 15}{36 \text{ games total}} = \frac{31}{36}.$$