Intro to Contemporary Math

Conditional Probability and Its Numerator

Department of Mathematics
UK

October 10, 2018

Announcement

- Homework due next Monday
- Exam 2 next Wednesday

Numerator of Conditional Probability

We have been using this formula today to compute conditional probability:

$$P(F|E) = \frac{\text{Number of outcomes in } E \bigcap F}{\text{Number of outcomes in } E}.$$

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Sometimes, we might know $P(F|E)$ and the number of outcomes in E. We can use them to find the number of outcomes in $E \bigcap F$:

$$\begin{aligned} & (\text{Number of outcomes in E}) \times P(F|E) \\ = \quad & \text{Number of outcomes in } E \bigcap F \end{aligned}$$

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Reminder when multiplying a whole number and a fraction:

$$a \times \frac{b}{c} = \frac{a \times b}{c}.$$

Outcomes in an Intersection

A jar contains beads which are either blue or white, and either round or pointy. If the probability of drawing a blue bead given that it is round is 47/50, and there are 250 round beads in the jar, how many beads are blue and round?

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$$\begin{aligned}
\frac{\text{Number of blue AND round beads}}{\text{Number of round beads}} &= \frac{47}{50} \\
\frac{\text{Number of blue AND round beads}}{\text{250 round beads}} &= \frac{47}{50} \\
\text{Number of blue AND round beads} &= 250 \cdot \frac{47}{50} \\
&= \frac{11,750}{50} = 235.
\end{aligned}$$

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$$\frac{\text{Number of blue AND round beads}}{\text{250 round beads}} = \frac{47}{50}$$

$$\begin{aligned}\text{Number of blue AND round beads} &= 250 \cdot \frac{47}{50} \\ &= \frac{11,750}{50} = 235.\end{aligned}$$

Check using decimals:

$$\frac{\text{Number of blue AND round beads}}{\text{Number of round beads}} = \frac{235}{250} = 0.94;$$

$$\frac{47}{50} = 0.94 \checkmark$$

?(8.1) Outcomes in an Intersection

A jar contains beads which are either blue or white, and either round or pointy. If the probability of drawing a round bead given that it is blue is 1/16, and the jar has 320 blue beads and 480 round beads, how many beads are blue and round?

$$\begin{aligned}
\frac{\text{Number of blue AND round beads}}{\text{Number of blue beads}} &= \frac{1}{16} \\
\frac{\text{Number of blue AND round beads}}{\text{320 blue beads}} &= \frac{1}{16} \\
\text{Number of blue AND round beads} &= 320 \cdot \frac{1}{16} \\
&= \frac{320}{16} = 20.
\end{aligned}$$

Check using decimals:

$$\frac{\text{Number of blue AND round beads}}{\text{Number of blue beads}} = \frac{20}{320} = 0.625;$$

$$\frac{1}{16} = 0.625 \checkmark$$

Fill in the Table

A jar contains beads which are either blue or white, and either round or pointy. There are 200 beads total, with 40 of them being pointy.

	Round	Pointy	Total
Blue			
White			
Total			200

Fill in the table. Suppose that:

- $P(\text{Bead is White} \mid \text{Bead is Round}) = 1/8$.
- $P(\text{Bead is Blue} \mid \text{Bead is Pointy}) = 3/20$.

Fill in the Table

A jar contains beads which are either blue or white, and either round or pointy. There are 200 beads total, with 40 of them being pointy.

	Round	Pointy	Total
Blue			
White			
Total		40	200

Fill in the table. Suppose that:

- $P(\text{Bead is White} \mid \text{Bead is Round}) = 1/8$.
- $P(\text{Bead is Blue} \mid \text{Bead is Pointy}) = 3/20$.

Fill in the Table (Round Bead Total)

A jar contains beads which are either blue or white, and either round or pointy. There are 200 beads total, with 40 of them being pointy.

	Round	Pointy	Total
Blue			
White			
Total	160	40	200

- 40 beads are pointy, so there must be

$$200 - 40 = 160$$

round beads.

Fill in the Table (White Round Beads)

	Round	Pointy	Total
Blue			
White			
Total	160	40	200

- We have $P(\text{Bead is White} \mid \text{Bead is Round}) = 1/8$. By definition,

$$\frac{\text{Number of white round beads}}{\text{Number of round beads}} = P(\text{White}|\text{Round})$$

$$\frac{\text{Number of white round beads}}{160} = \frac{1}{8}$$

	Round	Pointy	Total
Blue			
White	20		
Total	160	40	200

- We have $P(\text{Bead is White} \mid \text{Bead is Round}) = 1/8$. By definition,

$$\begin{aligned}
\frac{\text{Number of white round beads}}{\text{Number of round beads}} &= P(\text{White}|\text{Round}) \\
\frac{\text{Number of white round beads}}{160} &= \frac{1}{8} \\
\text{Number of white round beads} &= \frac{1}{8} \cdot 160 = \frac{160}{8} = 20.
\end{aligned}$$

?(8.2) Fill in the Table (Blue Pointy Beads)

	Round	Pointy	Total
Blue			
White	20		
Total	160	40	200

We know $P(\text{Bead is Blue} \mid \text{Bead is Pointy}) = 3/20$. Use this to find the number of blue pointy beads.

Fill in the Table (Blue Pointy Beads)

	Round	Pointy	Total
Blue		6	
White	20		
Total	160	40	200

We have $P(\text{Bead is Blue} \mid \text{Bead is Pointy}) = 3/20$. By definition,

$$\frac{\text{Number of blue pointy beads}}{\text{Number of pointy beads}} = P(\text{Blue}|\text{Pointy})$$

$$\frac{\text{Number of blue pointy beads}}{40} = \frac{3}{20}$$

$$\text{Number of blue pointy beads} = \frac{3}{20} \cdot 40 = \frac{3 \cdot 40}{20} = \frac{120}{20} = 6.$$

Fill in the Table (White Pointy Beads)

	Round	Pointy	Total
Blue		6	
White	20		
Total	160	40	200

Now use the totals to fill in missing counts.

- The number of white pointy beads is the number of pointy beads minus the blue pointy beads:

Fill in the Table (White Pointy Beads)

	Round	Pointy	Total
Blue		6	
White	20	34	
Total	160	40	200

Now use the totals to fill in missing counts.

- The number of white pointy beads is the number of pointy beads minus the blue pointy beads:

$$40 - 6 = 34.$$

?(8.3) Fill in the Table (Blue Round Beads)

	Round	Pointy	Total
Blue		6	
White	20	34	
Total	160	40	200

How many blue round beads are there?

Fill in the Table (Blue Round Beads)

	Round	Pointy	Total
Blue	140	6	
White	20	34	
Total	160	40	200

- The number of blue round beads is the number of round beads minus the white round beads:

$$160 - 20 = 140.$$

Fill in the Table (White Bead Total)

	Round	Pointy	Total
Blue	140	6	
White	20	34	
Total	160	40	200

Finally, fill in the missing totals.

- The number of white beads is the sum of the number of white round and white pointy beads:

Fill in the Table (White Bead Total)

	Round	Pointy	Total
Blue	140	6	
White	20	34	54
Total	160	40	200

Finally, fill in the missing totals.

- The number of white beads is the sum of the number of white round and white pointy beads:

$$20 + 34 = 54.$$

?(8.4) Fill in the Table (Blue Bead Total)

	Round	Pointy	Total
Blue	140	6	
White	20	34	54
Total	160	40	200

How many blue beads are there?

Fill in the Table (Blue Bead Total)

	Round	Pointy	Total
Blue	140	6	146
White	20	34	54
Total	160	40	200

- The number of blue beads is the sum of the number of blue round and blue pointy beads:

$$140 + 6 = 146.$$

Next Time

We will look at conditional probability with intervals.