Intro to Contemporary Math

Conditional Probability for Intervals

Department of Mathematics
UK

Announcements

- A homework assignment is due next Monday.
- Exam 2 is next Wednesday.

Continuous Probability Reminders

Use continuous probability when picking random real numbers.

- Sample spaces and events are made up of intervals.
- The length of an interval is the right endpoint minus the left endpoint.
-
-

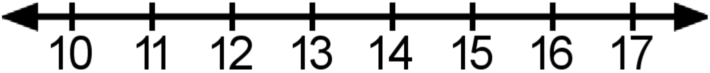
Continuous Probability Reminders

Use continuous probability when picking random real numbers.

- Sample spaces and events are made up of intervals.
- The length of an interval is the right endpoint minus the left endpoint.
- The probability of an interval event E is the length of E divided by the length of the sample space.
- The intersection of two intervals is the interval formed by their overlap.

Continuous Probability Review

Consider the sample space $\Omega = [10, 17]$ and event (interval) $F = [13, 16]$:



Continuous Probability Review

Consider the sample space $\Omega = [10, 17]$ and event (interval) $F = [13, 16]$:

-
-
-

Continuous Probability Review

Consider the sample space $\Omega = [10, 17]$ and event (interval) $F = [13, 16]$:

- The sample space has length $17 - 10 = 7$.
-
-

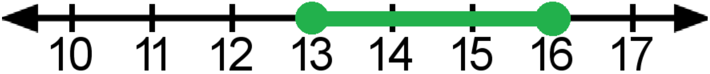
Continuous Probability Review

Consider the sample space $\Omega = [10, 17]$ and event (interval) $F = [13, 16]$:

- The sample space has length $17 - 10 = 7$.
- Event F has length $16 - 13 = 3$.
-

Continuous Probability Review

Consider the sample space $\Omega = [10, 17]$ and event (interval) $F = [13, 16]$:



- The sample space has length $17 - 10 = 7$.
- Event F has length $16 - 13 = 3$.
- Hence the probability of F is

$$\frac{\text{Length of } F}{\text{Total length}}$$

Continuous Probability Review

Consider the sample space $\Omega = [10, 17]$ and event (interval) $F = [13, 16]$:

- The sample space has length $17 - 10 = 7$.
- Event F has length $16 - 13 = 3$.
- Hence the probability of F is

$$\frac{\text{Length of } F}{\text{Total length}} = \frac{3}{7}.$$

Notice that F takes up 3/7ths of the total length of the sample space.

Conditional Probability for Intervals

Let E and F be events in a sample space Ω. Then the probability of event F given that E occurred is

$$P(F|E) = \frac{\text{Length of } E \bigcap F}{\text{Length of } E}.$$

Conditional Probability for Intervals (Details)

Let E and F be events in a sample space Ω. We have seen that

$$P(F|E) = \frac{P(E \bigcap F)}{P(E)}.$$

In terms of lengths, we have

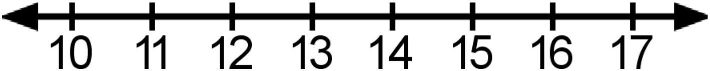
$$P(F|E) = \frac{\dfrac{\text{Length of } E \bigcap F}{\text{Total length}}}{\dfrac{\text{Length of } E}{\text{Total length}}},$$

and this simplifies to the fraction

$$P(F|E) = \frac{\text{Length of } E \bigcap F}{\text{Length of } E}.$$

Conditional Probability for Intervals 1

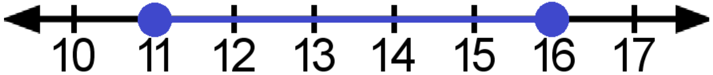
Let $\Omega = [10, 17]$, $E = [11, 16]$, and $F = [12, 15]$.
Let us compute $P(F|E)$.



- Find $E \bigcap F$ and its length:

Conditional Probability for Intervals 1

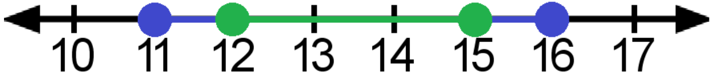
Let $\Omega = [10, 17]$, $E = [11, 16]$, and $F = [12, 15]$.
Let us compute $P(F|E)$.



- Find $E \bigcap F$ and its length:

Conditional Probability for Intervals 1

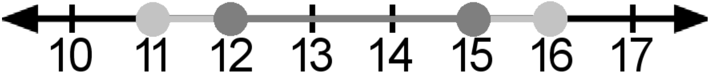
Let $\Omega = [10, 17]$, $E = [11, 16]$, and $F = [12, 15]$.
Let us compute $P(F|E)$.



- Find $E \bigcap F$ and its length:

Conditional Probability for Intervals 1

Let $\Omega = [10, 17]$, $E = [11, 16]$, and $F = [12, 15]$.
Let us compute $P(F|E)$.



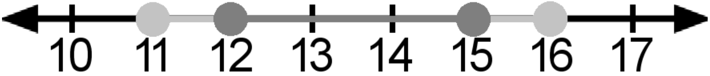
- Find $E \bigcap F$ and its length:

$$E \text{ and } F \text{ overlap on } [12, 15],$$

  which has length $15 - 12 = 3$.
-
-

Conditional Probability for Intervals 1

Let $\Omega = [10, 17]$, $E = [11, 16]$, and $F = [12, 15]$.
Let us compute $P(F|E)$.



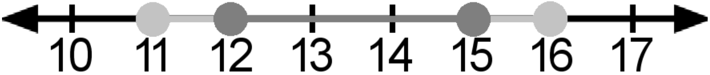
- Find $E \bigcap F$ and its length:

  $$E \text{ and } F \text{ overlap on } [12, 15],$$

  which has length $15 - 12 = 3$.
- Length of E is $16 - 11 = 5$.
-

Conditional Probability for Intervals 1

Let $\Omega = [10, 17]$, $E = [11, 16]$, and $F = [12, 15]$.
Let us compute $P(F|E)$.



- Find $E \bigcap F$ and its length:

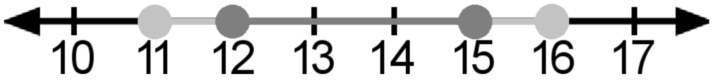
$$E \text{ and } F \text{ overlap on } [12, 15],$$

  which has length $15 - 12 = 3$.
- Length of E is $16 - 11 = 5$.
- Hence

$$P(F|E) = \frac{\text{Length of } E \bigcap F}{\text{Length of } E}$$

Conditional Probability for Intervals 1

Let $\Omega = [10, 17]$, $E = [11, 16]$, and $F = [12, 15]$.
Let us compute $P(F|E)$.



- Find $E \bigcap F$ and its length:

$$E \text{ and } F \text{ overlap on } [12, 15],$$

  which has length $15 - 12 = 3$.
- Length of E is $16 - 11 = 5$.
- Hence

$$P(F|E) = \frac{\text{Length of } E \bigcap F}{\text{Length of } E} = \frac{3}{5}.$$

  Notice that $E \bigcap F$ takes up 3/5ths of the total length of E.

?(9.1) Conditional Probability Practice 1

Let $\Omega = [24, 47]$, $E = [29, 43]$, and $F = [34, 38]$. Compute $P(F|E)$.

Hints:

1. Identify the intersection of $[29, 43]$ and $[34, 38]$ as an interval.
2. What is the length of the intersection?
3. What is the length of the given event?
4. Answer the question by dividing the appropriate lengths.

Conditional Probability Practice 1

Let $\Omega = [24, 47]$, $E = [29, 43]$, and $F = [34, 38]$. Compute $P(F|E)$.

Find $E \bigcap F$ and its length:

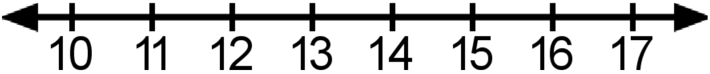
$$E \text{ and } F \text{ overlap on } [34, 38],$$

which has length $38 - 34 = 4$.
E itself has length $43 - 29 = 14$. Hence

$$P(F|E) = \frac{\text{Length of } E \bigcap F}{\text{Length of } E} = \frac{4}{14}.$$

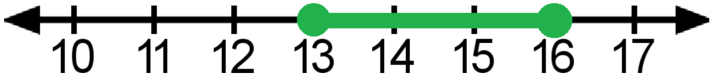
Conditional Probability for Intervals 2

Now let $\Omega = [10, 17]$, $E = [11, 15]$, and $F = [13, 16]$. Find $P(F|E)$.



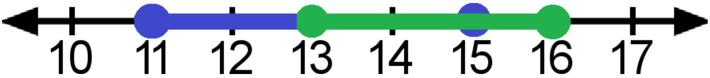
Conditional Probability for Intervals 2

Now let $\Omega = [10, 17]$, $E = [11, 15]$, and $F = [13, 16]$. Find $P(F|E)$.



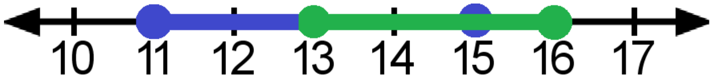
Conditional Probability for Intervals 2

Now let $\Omega = [10, 17]$, $E = [11, 15]$, and $F = [13, 16]$. Find $P(F|E)$.



Conditional Probability for Intervals 2

Now let $\Omega = [10, 17]$, $E = [11, 15]$, and $F = [13, 16]$. Find $P(F|E)$.



- Find $E \bigcap F$ and its length:

Conditional Probability for Intervals 2

Now let $\Omega = [10, 17]$, $E = [11, 15]$, and $F = [13, 16]$. Find $P(F|E)$.

- Find $E \bigcap F$ and its length:

$$E \bigcap F = [13, 15], \text{ so its length is } 15 - 13 = 2.$$

-
-

Conditional Probability for Intervals 2

Now let $\Omega = [10, 17]$, $E = [11, 15]$, and $F = [13, 16]$. Find $P(F|E)$.

- Find $E \bigcap F$ and its length:

$$E \bigcap F = [13, 15], \text{ so its length is } 15 - 13 = 2.$$

- Length of E is $15 - 11 = 4$.
-

Conditional Probability for Intervals 2

Now let $\Omega = [10, 17]$, $E = [11, 15]$, and $F = [13, 16]$. Find $P(F|E)$.

- Find $E \bigcap F$ and its length:

$$E \bigcap F = [13, 15], \text{ so its length is } 15 - 13 = 2.$$

- Length of E is $15 - 11 = 4$.
- Compute $P(F|E)$ using lengths:

$$P(F|E) = \frac{\text{Length of } E \bigcap F}{\text{Length of } E} = \frac{2}{4}.$$

?(9.2) Conditional Probability Practice 2

Let $\Omega = [47, 78]$, $E = [51, 60]$, and $F = [54, 63]$. Compute $P(F|E)$.

Hints:

1. Identify the intersection of $[51, 60]$ and $[54, 63]$ as an interval.
2. What is the length of the intersection?
3. What is the length of the given event?
4. Answer the question by dividing the appropriate lengths.

?(9.2) Conditional Probability Practice 2

Let $\Omega = [47, 78]$, $E = [51, 60]$, and $F = [54, 63]$. Compute $P(F|E)$.

Hints:

1. Identify the intersection of $[51, 60]$ and $[54, 63]$ as an interval.
2. What is the length of the intersection?
3. What is the length of the given event?
4. Answer the question by dividing the appropriate lengths.

Conditional Probability Practice 2

Let $\Omega = [47, 78]$, $E = [51, 60]$, and $F = [54, 63]$. Compute $P(F|E)$.

Find $E \bigcap F$ and its length:

$$E \text{ and } F \text{ overlap on } [54, 60],$$

which has length $60 - 54 = 6$.
Since E has length $60 - 51 = 9$,

$$P(F|E) = \frac{\text{Length of } E \bigcap F}{\text{Length of } E} = \frac{6}{9}.$$