Intro to Contemporary Math

Majority Criterion

Nicholas Nguyen
nicholas.nguyen@uky.edu

Department of Mathematics
UK

Agenda

- Majority Criterion
- Testing Voting Methods

WebWork

- Homework 2 is due next Monday.

Majority Criterion

Majority Criterion (MA criterion):
A candidate who **has over 50% of the first place votes** (a majority candidate) **should win**.

- Do all voting methods comply with (obey) this criterion (law)?

Not the Majority Criterion

- The MA criterion **does not** demand that every election have a majority candidate.
- Fairness criteria only **put demands on voting methods**, not on voters.

Majority Criterion Compliance

A voting method:

- Always satisfies the MA criterion if any election with a majority candidate ends in that candidate winning
- Can violate the MA criterion if we can find (or make) an election with a majority candidate who loses

Testing for MA Violation

Goal: See if a **majority candidate can lose** with a voting method (catch it in the act of not choosing a majority candidate as a winner)

To test if a voting method can violate (fail) MA,

1) Find or make an election
2) Check if there is a majority candidate
3) Determine the winner using the voting method

Testing for MA Violation

1) Find or make an election
2) Check if there is a majority candidate
3) Determine the winner using the voting method

Outcomes/Results:

- **Inconclusive:** No majority candidate, or majority candidate wins (with the voting method)
- **Violation:** There is a majority candidate, but the voting method picks someone else as the winner.

Testing for MA Violation Outcomes

- **Inconclusive:** No majority candidate, or majority candidate wins (with the voting method)
- **Violation:** There is a majority candidate, but the voting method picks someone else as the winner.

If a voting method can violate the MA criterion, it means that a majority candidate **can** lose with the voting method.

MA Testing and Borda Count (1)

Let's see if the Borda count method can violate the majority criterion.

Let's try this election:

2	2	1
A	**B**	**C**
B	**C**	**B**
C	**A**	**A**

Point totals:

A gets 9 points

B gets 12 points

C gets 9 points

-
-

MA Testing and Borda Count (1)

Let's see if the Borda count method can violate the majority criterion.

Let's try this election:

2	2	1
A	**B**	**C**
B	**C**	**B**
C	**A**	**A**

Point totals:
A gets 9 points
B gets 12 points
C gets 9 points

- No majority candidate
- B wins with Borda count

MA Testing and Borda Count (1)

Let's see if the Borda count method can violate the majority criterion.

Let's try this election:

2	2	1
A	**B**	**C**
B	**C**	**B**
C	**A**	**A**

Point totals:
A gets 9 points
B gets 12 points
C gets 9 points

- No majority candidate
- B wins with Borda count

Testing for a violation with this election gives an **inconclusive result**, because there is **no majority candidate**.

Let's see if the Borda count method can violate the majority criterion.

Let's try this election:

2	2	1
A	**B**	**C**
B	**C**	**B**
C	**A**	**A**

Point totals:
A gets 9 points
B gets 12 points
C gets 9 points

- No majority candidate
- B wins with Borda count

Testing for a violation with this election gives an **inconclusive result**, because there is no majority candidate.

This election will not help us determine if Borda count can violate the majority criterion.

MA Testing and Borda Count (2)

2	3
A	**B**
B	**C**
C	**A**

Point totals:
A gets 9 points
B gets 13 points
C gets 8 points

-
-

MA Testing and Borda Count (2)

2	3
A	**B**
B	**C**
C	**A**

Point totals:
A gets 9 points
B gets 13 points
C gets 8 points

- B is a majority candidate with 3/5 first place votes
- B wins with Borda count

MA Testing and Borda Count (2)

2	3
A	**B**
B	**C**
C	**A**

Point totals:
A gets 9 points
B gets 13 points
C gets 8 points

- B is a majority candidate with 3/5 first place votes
- B wins with Borda count

Testing for a violation with this election gives an **inconclusive result**, because the **majority candidate is B, who also won** with the voting method Borda count.

2	3
A	**B**
B	**C**
C	**A**

Point totals:
A gets 9 points
B gets 13 points
C gets 8 points

- B is a majority candidate with 3/5 first place votes
- B wins with Borda count

Testing for a violation with this election gives an **inconclusive result**, because the **majority candidate is B, who also won** with the voting method Borda count.
The reason why **this single example does not show satisfaction** is because **we do not know whether a violation could occur in a different election** or not.

MA Testing and Borda Count (3)

3	2
A	**B**
B	**C**
C	**A**

Point totals:
A gets 11 points
B gets 12 points
C gets 7 points

-
-

MA Testing and Borda Count (3)

3	2
A	**B**
B	**C**
C	**A**

Point totals:
A gets 11 points
B gets 12 points
C gets 7 points

- A is a majority candidate with 3/5 first place votes, but
- someone else, B, wins with Borda count!

MA Testing and Borda Count (3)

3	2
A	**B**
B	**C**
C	**A**

Point totals:
A gets 11 points
B gets 12 points
C gets 7 points

- A is a majority candidate with 3/5 first place votes, but
- someone else, B, wins with Borda count!

The Borda Count method **can violate** the Majority criterion!

MA Testing and Borda Count (3)

3	2
A	**B**
B	**C**
C	**A**

Point totals:
A gets 11 points
B gets 12 points
C gets 7 points

- A is a majority candidate with 3/5 first place votes, but
- someone else, B, wins with Borda count!

The Borda Count method **can violate** the Majority criterion!
We have **found** an election where there is a majority candidate, A, but the Borda count method picked B instead as the winner.

MA Testing and Borda Count

- Election 1 is **inconclusive**: with no majority candidate, it won't help us learn if they always win or can lose.
-
  -
-

MA Testing and Borda Count

- Election 1 is **inconclusive**: with no majority candidate, it won't help us learn if they always win or can lose.
- Election 2 is **inconclusive**: the majority candidate won this time, but we do not know if they always win or can lose.
  -
-

MA Testing and Borda Count

- Election 1 is **inconclusive**: with no majority candidate, it won't help us learn if they always win or can lose.
- Election 2 is **inconclusive**: the majority candidate won this time, but we do not know if they always win or can lose.
  - It's not satisfaction: we do not know if they always win from this election.
-

MA Testing and Borda Count

- Election 1 is **inconclusive**: with no majority candidate, it won't help us learn if they always win or can lose.
- Election 2 is **inconclusive**: the majority candidate won this time, but we do not know if they always win or can lose.
  - It's not satisfaction: we do not know if they always win from this election.
- Election 3 shows a **violation**: the majority candidate lost!

MA Testing and PwE

We used Plurality with Elimination on this election last week:

2	3	6
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

In this election,

- Cobalt is the majority candidate with 6/11 first place votes.
- Cobalt wins with PwE automatically, due to having over 50% of the first place votes at the beginning.

Testing for a violation with this election gives an **inconclusive** result, because the majority candidate is Cobalt, who also won with the voting method PwE.

2	3	6
Azure	Blue	Cobalt
Blue	Cobalt	Blue
Cobalt	Azure	Azure

In this election,

- Cobalt is the majority candidate with 6/11 first place votes.
- Cobalt wins with PwE automatically, due to having over 50% of the first place votes at the beginning.

Testing for a violation with this election gives an **inconclusive** result, because the majority candidate is Cobalt, who also won with the voting method PwE.

The reason why this **single example** does not show satisfaction is because we do not know whether a violation could occur in a different election or not.

?(6.1) MA Testing

Below are 3 elections using an unknown voting method:

Election 1:

2	3	4
C	**A**	**B**
B	**B**	**A**
A	**C**	**C**

Winner: B

Election 2:

2	3	4
A	**A**	**B**
B	**C**	**A**
C	**B**	**C**

Winner: B

Election 3:

2	3	6
A	**A**	**B**
B	**C**	**A**
C	**B**	**C**

Winner: B

Which election shows that the voting method can violate the MA criterion?
Press the number of the election that **shows a violation** (1-3).

MA Testing

- Election 2 shows a **violation** of the MA criterion:
  - Candidate A was the majority candidate (5/9 first place votes), but
  - Candidate B was declared the winner
- Election 1 gives an **inconclusive** result because it did not have a majority candidate
- Election 3 gives an **inconclusive** result because it has a majority candidate (B) that won

Testing for MA Satisfaction

To test if a voting method satisfies (passes) MA, we must:
1) Study the rules of the voting method
2) Determine if a majority candidate is guaranteed to win under the rules
If a majority candidate is **guaranteed to win**, the voting method **satisfies the MA criterion.**

MA Satisfaction Testing and PwE

Let's look at the rules for Plurality with Elimination:

Step 1: Check for a candidate with **over** 50% of the first place votes.
If there is one, that candidate is the **winner**.
If not, go to **Step 2**.

Step 2: **Eliminate** the candidate with the fewest first place votes. Remove them from the schedule.

Step 3: **Push** the other candidates up to fill in the blanks. Then **go back to Step 1.**

- If we have an election with a majority candidate, then that candidate would have over 50% of the first place votes at the beginning.

Let's look at the rules for Plurality with Elimination:

Step 1: Check for a candidate with **over** 50% of the first place votes.
If there is one, that candidate is the **winner**.
If not, go to **Step 2**.

Step 2: **Eliminate** the candidate with the fewest first place votes. Remove them from the schedule.

Step 3: **Push** the other candidates up to fill in the blanks. Then **go back to Step 1.**

- If we have an election with a majority candidate, then that candidate would have over 50% of the first place votes at the beginning.
  According to Step 1, the **majority candidate would always win** right away, and the process would never reach Step 2. Hence PwE **always satisfies** MA.

?(6.2) MA Satisfaction: Plurality

The candidate with the most first place votes wins.

- Does Plurality satisfy MA?
  Type "y" or "n" on your device.
- Hint: if a candidate has over 50% of the first place votes, then are they guaranteed to have the most first place votes?

MA Satisfaction: Plurality

Yes: If there is a majority candidate, it would have over 50% of the first place votes at the beginning. No other candidate can have more first place votes than a majority candidate (they can only get less than 50% of the first place votes). Hence a majority candidate will always win with Plurality, so Plurality always satisfies MA.

MA Satisfaction: Pairwise Comparison

To check if Pairwise Comparison satisfies or violates MA, let us work with:

- An election with N candidates
- A majority candidate named A

Let us see if A wins, or how A could lose.

?(6.3) MA and 1-on-1 Comparisons

What happens to a majority candidate like A here in a comparison against any opponent B?

over half	less than half
A	B? A?
B	A? B?

A) A wins the comparison
B) B wins the comparison
C) There is a tie

MA and 1-on-1 Comparisons

over half	less than half
A	B? A? A? B?
B	

More than half of the voters rank A first, so these voters rank A higher than B and go in A's camp. Since A's camp already has over half of the voters, there is no way B can get a bigger camp, so A wins.

?(6.4) MA Satisfaction: Pairwise Comparison

More than half of the voters rank A first, so these voters rank A higher than B and go in A's camp. Since A's camp already has over half of the voters, there is no way B can get a bigger camp, so A wins.

Thus, the majority candidate A wins all of its comparisons.

Will A win the overall election with Pairwise Comparison?

Type "y" or "n" on your device.

Hint: To win with Pairwise Comparison, a candidate must get the most points from winning or tying comparisons.

MA Satisfaction: Pairwise Comparison

Yes: If A wins its comparisons against all $N-1$ opponents, then A must have the most points, $N-1$. No one else can get $N-1$ points because they lost to A and did not get a point from their comparison vs. A. They can only get up to $N-2$ points against the other opponents.
Since A has the most points, it wins with Pairwise Comparison.
Hence Pairwise Comparison always satisfies MA.

Majority Candidates and Condorcet Candidates

As we just saw, every majority candidate is also a Condorcet candidate.

- Not every Condorcet candidate is a majority candidate.

Next time

- We will introduce the Condorcet criterion: Condorcet candidates should win
- Homework 2 is due next Monday.