Intro to Contemporary Math

Monotonicity Criterion

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Agenda

- Monotonicity Criterion
- Testing Voting Methods

WebWork

- The fourth homework assignment (HW3) is due next Monday.
- Exam 1 is next Wednesday.

Today's Criterion

Monotonicity Criterion (MO):
Giving the winner more first-place votes by taking ballots and moving the winner up (without shuffling the other candidates) should not make the winner lose.

Testing for the Monotonicity Criterion

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To test if a voting method violates (fails) MO, we must:
1) Find an election using the voting method
2) Determine the winner using the voting method
3) Take some ballots from voters who chose a loser as their first choice, and change their ballot by moving the winner up. Shift the others down:

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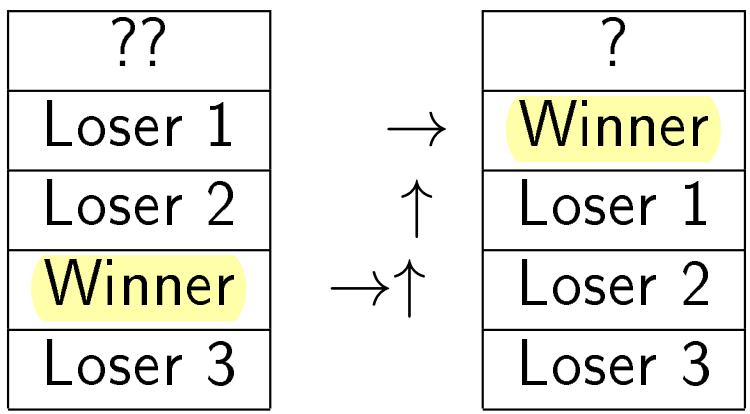

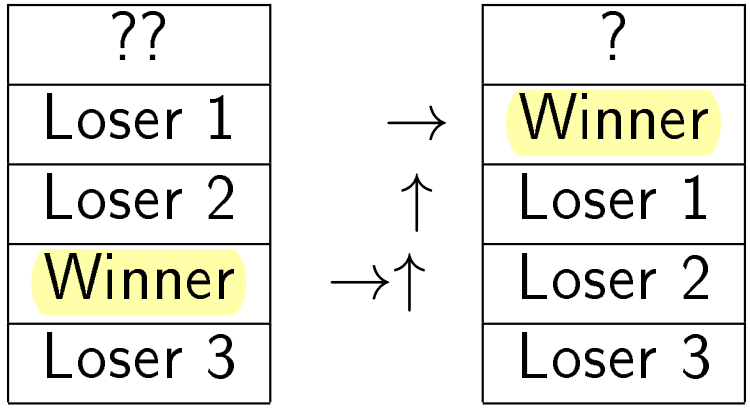

4) After changing ballots, determine the winner of this modified election.

- **Invalid:** Losers were shuffled or the winner moved down on changed ballots
- **Inconclusive:** Original winner is declared the winner in the modified election. Perhaps a different choice of ballots may cause a change.
- **Violation:** The original winner loses in the modified election.

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Testing for the Monotonicity Criterion

To test if a voting method satisfies (passes) MO, we must:
1) Study the rules of the voting method
2) Determine if a winner will not lose by getting more better ranking votes

If **a winner cannot lose by getting more better ranking votes**, the voting method **satisfies the MO criterion.**

?(8.1) MO Testing for Violations

Below is an election using an unknown voting method:

2	3	4
C	**A**	**B**
B	**B**	**C**
A	**C**	**A**

(Winner: B)

We will take one ballot from the second column and change it.
Which ballot change is allowed when testing MO?
Type and send a number.

Choice 1:

1
B
A
C

Choice 2:

1
B
C
A

Choice 3:

1
C
A
B

MO Testing for Violations

- Choice 1 is **valid** for testing for MO violations: B moved up, and the others are still in order.
- Choice 2 has B moved up, but it puts C above A. Originally, in the second column, A was above C. Ballot changes can only move the winner to the top and keep the order of the others. Choice 2 is **not a valid change.**
- In Choice 3, B got moved down, so it is **not a valid change.**

?(8.2) MO Satisfaction?

This election used Plurality:

1	4
C	A
B	B
A	C

→

1	4
A	A
C	B
B	C

Originally, A wins with Plurality. When testing for MO, we changed the C-B-A ballot in column 1 to an A-C-B ballot. After the change, **A still wins** with Plurality. Does this election show:

1) Plurality can violate MO
2) Plurality always satisfies MO
3) an inconclusive result

Type and send a number.

MO Satisfaction? Inconclusive

Inconclusive!

Just because A won after getting another first place vote does not mean that it always remains the winner when getting more first place votes from changing ballots! We have to look at the rules!

?(8.3) MO and Plurality

The candidate with the most first place votes wins.

- Does Plurality satisfy MO? Press 1 for Yes, 2 for No. Hint: The original winner had the most 1st place votes. What happens if they get more (from opponents) - will they still have the most?

MO and Plurality

Yes!

- Winner has the **most first place votes.** Getting more by changing ballots can only give the winner **more first place votes**, so they **still have the most. Plurality method always satisfies MO.**

?(8.4) MO and Borda

The candidate with the most Borda points wins. Higher rankings give more points.

- Does Borda satisfy MO? Press 1 for Yes, 2 for No.
  Hint: The original winner had the most Borda points. If ballots are changed, some of their lower ranking votes get raised, giving more points. Will they still have the most points?

MO and Borda

Yes!

- Winner has the **most Borda points.** Getting **better ranking votes will only give the winner more points** (and the losers less points), so they would still have the most points and win. **Borda Count satisfies MO.**

?(8.5) MO and Pairwise

The candidate with the most points from winning or tying comparisons wins.

- Does Pairwise comparison satisfy MO?
  Press 1 for Yes, 2 for No.
  Hint: The original winner had the most pairwise points. If ballots are changed, they will move above some opponents on the changed ballots. What effect does this have on comparisons? Will they still have the most points?

MO and Pairwise

Yes! Winner has the most points from winning comparisons. Performance in a comparison is based on being **above** opponents, so

- More ballots with the winner **above** others
- More voters in the winner's camps which means
- **More comparisons won or tied and more points** (while losers that beat the winner in a comparison may lose points after ballots are changed).
  Pairwise Comparison satisfies MO.

?(8.6) MO and PwE (Votes to Win)

5	6	6
Azure	Cobalt	Blue
Blue	Azure	Azure
Cobalt	Blue	Cobalt

How many 1st place votes are needed to win in PwE?
Type in a number.

MO and PwE (Votes to Win)

5	6	6
Azure	Cobalt	Blue
Blue	Azure	Azure
Cobalt	Blue	Cobalt

A candidate needs over 50% of the 1st place votes to win.
There are 17 voters.
Add one to get 18.
Divide by 2 to get 9, which rounds up to $\boxed{9}$ first place votes.
Need 9 out of 17 first place votes to win.

?(8.7) MO and PwE (Elimination)

5	6	6
Azure	Cobalt	Blue
Blue	Azure	Azure
Cobalt	Blue	Cobalt

Press the first letter of the color that gets eliminated first.

MO and PwE (Elimination)

5	6	6
Azure	Cobalt	Blue
Blue	Azure	Azure
Cobalt	Blue	Cobalt

Azure is eliminated (only 5 first place votes, the least of any color)

MO and PwE (Elimination)

5	6	6
	Cobalt	Blue
Blue		
Cobalt	Blue	Cobalt

Azure is eliminated (only 5 first place votes, the least of any color)

?(8.8) MO and PwE (Original Winner)

5	6	6
Blue	Cobalt	Blue
Cobalt	Blue	Cobalt

Winner is:

1) Blue
2) Cobalt
3) Tie

MO and PwE (Original Winner)

5	6	6
Blue	Cobalt	Blue
Cobalt	Blue	Cobalt

Blue gets 11 first place votes, which is more than 50% of the total number of voters. Blue wins.

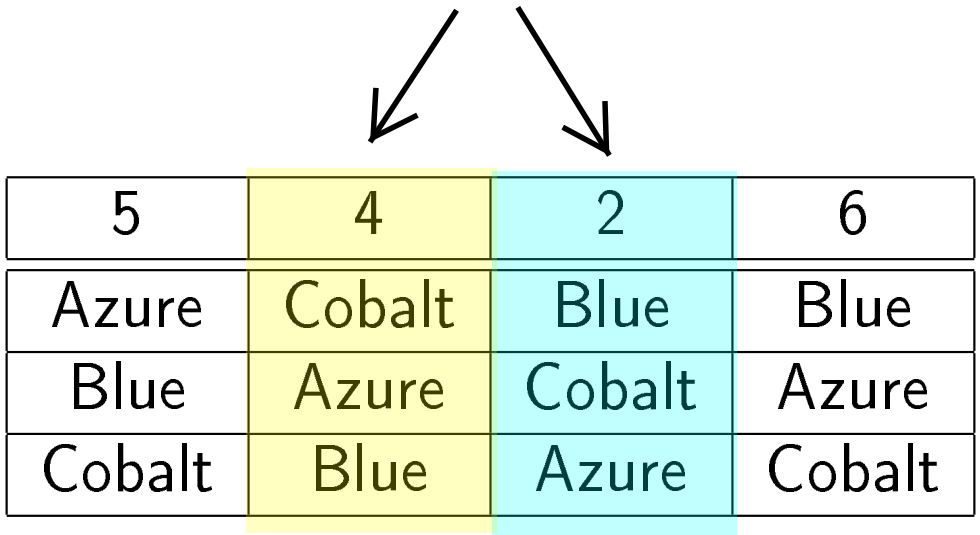
MO and PwE (Ballot Change!)

Change two ballots (out of six) in the second column so Blue is at top, followed by Cobalt, then Azure.

Before:

5	6	6
Azure	Cobalt	Blue
Blue	Azure	Azure
Cobalt	Blue	Cobalt

After:



5	4	2	6
Azure	Cobalt	Blue	Blue
Blue	Azure	Cobalt	Azure
Cobalt	Blue	Azure	Cobalt

MO and PwE (Ballot Change!)

Change two ballots (out of six) in the second column so Blue is at top, followed by Cobalt, then Azure.

After:

5	4	2	6
Azure	Cobalt	Blue	Blue
Blue	Azure	Cobalt	Azure
Cobalt	Blue	Azure	Cobalt

?(8.9) MO and PwE (New Elimination)

5	4	2	6
Azure	Cobalt	Blue	Blue
Blue	Azure	Cobalt	Azure
Cobalt	Blue	Azure	Cobalt

Press the first letter of the color to be eliminated.

MO and PwE (New Elimination)

5	4	2	6
Azure	Cobalt	Blue	Blue
Blue	Azure	Cobalt	Azure
Cobalt	Blue	Azure	Cobalt

Eliminate Cobalt this time (only 4 first place votes)

MO and PwE (New Elimination)

5	4	2	6
Azure		Blue	Blue
Blue	Azure		Azure
	Blue	Azure	

Eliminate Cobalt this time (only 4 first place votes)

?(8.10) MO and PwE (New Winner)

5	4	2	6
Azure	Azure	Blue	Blue
Blue	Blue	Azure	Azure

Winner is:

1) Azure
2) Blue
3) Tie

MO and PwE (New Winner)

Azure wins with 9 out of 17 votes.

5	4	2	6
Azure	Azure	Blue	Blue
Blue	Blue	Azure	Azure

?(8.11) MO Testing: PwE

- Original winner: Blue
- New winner: Azure

Does this election show:
1) PwE can violate MO
2) PwE always satisfies MO
3) an inconclusive result

MO Testing: PwE

Violation:

Blue originally won, but when we changed the ballots to give Blue two more 1st place votes, Azure won instead! The MO criterion says that is not supposed to happen (the law says Blue should have remained the winner), so this election shows that PwE can violate the MO criterion!

MO and PwE Explanation

- The two extra votes Blue got resulted in a second round of voting where it did not get any new votes.
- Another way: The two extra votes Blue got at the beginning made it lose the five votes it would have gotten from the second round in the original election.
  - Originally, Blue got 11 votes. In the modified election, Blue got 8. Blue lost 3 votes overall in the modified election compared to the original election.
- Changing first place votes can change what gets eliminated, which can result in a different winner based on who got more votes in later rounds.