

Intro to Contemporary Math

Voting Theory Review

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Agenda

- ▶ Violation and Satisfaction
- ▶ Voting Methods and Fairness Criteria Hints

Announcements

- ▶ Homework 3 is due tonight.
- ▶ Exam 1 is this Wednesday. Don't forget your supplies (pen or pencil and a calculator that is not a cell phone)

Violation and Satisfaction

- ▶ A **voting method** is a procedure for picking a winner from an election's ballots (preference schedule).
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- ▶ A voting method (always) **satisfies** a fairness criterion if the method always follows (obeys) that criterion.
This can only be *shown using the method's rules and steps*, not with an election.
- ▶ A voting method can **violate** a fairness criterion if there is an election where the method breaks (disobeys) the criterion.
This must be *shown in an election*.

Summary of Compliance

Method	MA	CO	MO	IIA
Plurality	Satisfy	Violate	Satisfy	Violate
Borda	Violate	Violate	Satisfy	Violate
PwE.	Satisfy	Violate	Violate	Violate
Pairwise	Satisfy	Satisfy	Satisfy	Violate

Hints:

- ▶ Only Borda Count can violate MA.



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Hints:

- ▶ Only Borda Count can violate MA.
- ▶ Only Pairwise Comparison always satisfies CO.
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Hints:

- ▶ Only Borda Count can violate MA.
- ▶ Only Pairwise Comparison always satisfies CO.
- ▶ Only PwE can violate MO.
- ▶ All four can violate IIA.

Summary

- ▶ The whole point behind fairness criteria is to not only **state** what a fair election should be like, but allow us to **test** whether a voting method satisfies or violates the criterion.

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- ▶ The majority criterion and Condorcet criterion involve candidates who are popular in some way:
 - ▶ The majority criterion wants a candidate **who has over 50% of the first place votes to win.**
 - ▶ The Condorcet criterion wants a candidate **who wins all their pairwise comparisons to win.**

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- ▶ The IIA criterion wants to **keep losers who quit (or are otherwise removed) from affecting the election results.**

No Perfect Method

Theorem (Arrow's Impossibility Theorem)

*It is **impossible** for a voting method to satisfy all four fairness criteria.*

Exam Format

Two parts:

- ▶ 16 Multiple Choice questions
 - ▶ No partial credit
 - ▶ Record answers on front page of exam
- ▶ Short Answer (SA) questions
 - ▶ Length of SA is about same as mini-exam

What to Expect

- ▶ Know how the various voting methods work.
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What to Expect

- ▶ Know how the various voting methods work.
- ▶ Know what the fairness criteria are and how to test for violations.
- ▶ For each criterion, know what it means to violate or always satisfy it.

Next time

- ▶ The exam is this Wednesday. Bring a calculator that is not a cell phone.
- ▶ Homework 3 is due tonight.