# Intro to Contemporary Math

## Probability Theory Introduction

Department of Mathematics
UK

# Announcements

- Exam 1 will be returned next Monday. Grades will be posted on Sunday.
- You have a new homework assignment. It is due next Monday.

# Topic Idea: Probability

### Definition

The **probability** of an event is a measurement of its likelihood to occur.

There are two interpretations of probability: **experimental** and **theoretical**.

# Fractions

In both interpretations, we use fractions to express portions of quantities, and in general, quantities which are not whole numbers.

## Example

The fraction

$$\frac{2}{5}, \text{ or } 2/5$$

means two parts out of five total.

# Fractions

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## Example

The fraction

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means two parts out of five total.
Fractions can be converted to decimals by dividing:

$$2/5 = 0.4,$$

but we will use fractions for better accuracy.

# Experimental Probability

Perform an experiment over and over, and divide the number of times a desired event occurs by the total number of times the experiment was performed.

## Example

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## Example

We could toss a coin 100 times. If it came up heads 47 times, we would have measured that the probability of flipping this coin and getting heads is

$$\frac{\text{number of heads}}{\text{total tosses}} = \frac{47}{100}.$$

# Theoretical Probability

Suppose an experiment can end in $n$ ways. If the results cannot be told apart except by name, we assume they are equally-likely, and assign each result a probability of

$$\frac{1}{n}.$$

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## Example

The coin can land in 1 of 2 ways: heads or tails. If we assume the coin is equally-likely to land heads or tails, then the probability of each result is

$$\frac{\textbf{One}\text{ side is heads}}{\textbf{Two}\text{ sides total}} = \frac{1}{2}.$$

# Outcomes and Events

## Definitions

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A (chance) experiment is a procedure whose result can be one out of many possibilities.

- Each possible result is called an **outcome**.
- An **event** is any particular outcome or group of outcomes.
- The **sample space** is a list of all possible outcomes.

# Six-Sided Die

If we roll a six-sided die, one numbered side will be on top of the die when it lands.

- There are a total of 6 outcomes, one for each side.
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- There are a total of 6 outcomes, one for each side.
- Example of an event: "We roll an odd number." The outcomes "1," "3," and "5" are described by this event.
- The sample space is a list of all sides:

$$\{1, 2, 3, 4, 5, 6\}.$$

# Computing Probability

Given that all outcomes are equally-likely, the probability of an event, $P(\text{"Event"})$, is

$$P(\text{"Event"}) = \frac{\text{Number of outcomes described by the event}}{\text{Total number of outcomes}}$$

In other words, it is the ratio of outcomes in the event compared to the total amount of outcomes.

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In other words, it is the ratio of outcomes in the event compared to the total amount of outcomes.
**Note**: The probability of an event must be a number between 0 and 1.
An event with a probability of 0 is impossible.
An event with a probability of 1 is certain.

# Six-Sided Die

- The event "We roll a 5" describes one outcome (5 on top), so

$$P(\text{"We roll a 5"}) = \frac{\text{One side with a 5}}{\text{Six sides total}} = \boxed{\frac{1}{6}}.$$

# Six-Sided Die

- The event "We roll an odd number" describes three outcomes: 1, 3, or 5 on top. Thus

$$
\begin{aligned}
P(\text{"We roll an odd number"}) &= \frac{\text{Three sides in event}}{\text{Six sides total}} \\
&= \boxed{\frac{3}{6}}
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$$

# Six-Sided Die

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  The answer 3/6 can be reduced to 1/2, or to the decimal expression 0.5, but the original answer makes it easier to see the number of outcomes in the event and sample space.

# Six-Sided Die

- The event "We roll a 7" describes no outcomes (7 is not on the die), so

$$P(\text{"We roll a 7"}) = \frac{\text{No outcomes in event}}{\text{Six outcomes total}} = \boxed{\frac{0}{6}}, \text{ or } 0.$$

This event is impossible.

# Six-Sided Die

- The event "We roll a number" describes all six outcomes (all six sides have a number on them), so

$$P(\text{"We roll a number"}) = \frac{\text{Six outcomes in event}}{\text{Six outcomes total}} = \boxed{\frac{6}{6}}, \text{ or } 1.$$

This event is certain to occur.

# ?(1.1) Six-Sided Die

On a six-sided die, what is the probability of the event "We roll a number strictly greater than 2?"
Give a fraction as your answer.
You do not need to reduce your answer.

# Six-Sided Die

The event "We roll a number strictly greater than 2" describes four outcomes: 3, 4, 5, or 6 on top. Thus

$$\frac{\text{Four outcomes in event}}{\text{Six outcomes total}} = \boxed{\frac{4}{6}} = \frac{2}{3}.$$

# ?(1.2) Marbles

A jar has 11 red marbles and 16 blue ones. If a marble is drawn at random, what is the probability of drawing a blue marble?

# Marbles

A jar has 11 red marbles and 16 blue ones. If a marble is drawn at random, what is the probability of drawing a blue marble? Our sample space is all the marbles in the jar:

$$\text{There are } 11+16=27 \text{ marbles total.}$$

Hence the probability of drawing a blue marble is

$$\frac{16 \text{ blue marbles}}{27 \text{ marbles total}} = \boxed{\frac{16}{27}}.$$

# Checking Answers

The probability of an event is a number between 0 and 1.

- The number of outcomes in an event (numerator) cannot be negative and it cannot be more than the total number of outcomes (denominator).
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# Checking Answers

The probability of an event is a number between 0 and 1.

- The number of outcomes in an event (numerator) cannot be negative and it cannot be more than the total number of outcomes (denominator).
- You can convert to a decimal to see how big a fraction is.

Example: if you are computing the probability of an event and get an answer that converts to 1.11, it must be wrong: it is too big.

# ?(1.3) Numbers which can be Probabilities

Which of these numbers can be a probability of an event?

$3/2$

$4.6$

$-6/7$

$8/9$

$1.2$

$101\%$

# Numbers which can be Probabilities

Which of these numbers can be a probability of an event?

$$8/9 = \frac{\text{8 outcomes in the event}}{\text{9 outcomes total}}$$

The probability of any event must be a number between 0 and 1.

A number like $3/2$ cannot be a probability. No event can have 3 outcomes in a sample space with only 2.

# End

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