Intro to Contemporary Math

Complements, Unions, and Intersections of Events

Department of Mathematics

UK

Numbers in a Table

Announcement: You have homework due tonight.
Here is a list of 123 beads sorted by color and shape. Suppose we draw a bead.

	$\triangle$	$\square$	Total
Red	1	8	9
Green	64	16	80
Blue	2	32	34
Total	67	56	123

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Let E be the event "The bead is green," and F be the event "The bead is a square."

$64 + 16 = 80$ beads are green out of 123 total, so

$$P(E) = \boxed{\frac{80}{123}}.$$

?(2.1) Numbers in a Table (Event F)

Here is a list of 123 beads sorted by color and shape. Suppose we draw a bead.

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Total	67	56	123

Let E be the event "The bead is green," and F be the event "The bead is a square."
What is $P(F)$, the probability of drawing a square bead? Type and send a **fraction**.

Numbers in a Table (Event F)

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Red	1	8	9
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Total	67	56	123

Let E be the event "The bead is green," and F be the event "The bead is a square."

There are $8+16+32=56$ square beads out of 123 total, so

$$P(F)=\boxed{\frac{56}{123}}.$$

Complements

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$$P(\bar{E}) = 1 - P(E).$$

?(2.2) Complements Practice

- The complement of an event E is the event "E does not happen."
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- If we know the probability of E, $P(E)$, then

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An event E in a sample space with seven objects has probability $4/7$ of occurring. What fraction is $P(\bar{E})$?

?(2.2) Complements Practice

An event E in a sample space with seven objects has probability $4/7$ of occurring. What fraction is $P(\bar{E})$?

$$\begin{aligned} P(\bar{E}) &= 1 - P(E) \\ &= 1 - \frac{4}{7} \end{aligned}$$

To add or subtract a whole number and a fraction, use a common denominator:

$$\begin{aligned} &= 1 \cdot \frac{7}{7} - \frac{4}{7} \\ &= \frac{7}{7} - \frac{4}{7} \end{aligned}$$

Complements Practice

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To add or subtract a whole number and a fraction, use a common denominator:

$$\begin{aligned} &= 1 \cdot \frac{7}{7} - \frac{4}{7} \\ &= \frac{7}{7} - \frac{4}{7} \\ &= \frac{7-4}{7} = \boxed{\frac{3}{7}}. \end{aligned}$$

?(2.3) Numbers in a Table (Complement)

Here is a list of 123 beads sorted by color and shape. Suppose we draw a bead.

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Red	1	8	9
Green	64	16	80
Blue	2	32	34
Total	67	56	123

Let E be the event "The bead is green," and F be the event "The bead is a square."
Find $P(\bar{E})$.

Numbers in a Table (Complement)

	$\triangle$	$\square$	Total
Red	1	8	9
Green	64	16	80
Blue	2	32	34
Total	67	56	123

Let E be the event "The bead is green," and F be the event "The bead is a square."
Then $\bar{E}$ is the event "The bead is not green," so it is red or blue:

$$9 + 34 = 43 \text{ beads are not green.}$$

$$P(\bar{E}) = \boxed{\frac{43}{123}}.$$

Numbers in a Table (Complement)

	$\triangle$	$\square$	Total
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Total	67	56	123

Let E be the event "The bead is green," and F be the event "The bead is a square."
Using the complement, we see that 80 beads are green out of 123 total, so

$$P(\bar{E}) = 1 - P(E) = 1 - \frac{80}{123} = \frac{123}{123} - \frac{80}{123} = \boxed{\frac{43}{123}}.$$

Unions and Intersections (Or and And)

Let E and F be two events in a sample space.

- The event that has outcomes in E, **or** in F, **or** in both, is called the **union** of E and F and is denoted $E \bigcup F$.
-

Unions and Intersections (Or and And)

Let E and F be two events in a sample space.

- The event that has outcomes in E, **or** in F, **or** in both, is called the **union** of E and F and is denoted $E \bigcup F$.
- The event that has outcomes in **both** E **and** F, is called the **intersection** of E and F and is denoted $E \bigcap F$.

Numbers in a Table (Or)

	△	□	Total
Red	1	8	9
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Blue	2	32	34
Total	67	56	123

Let E be the event "The bead is green," and F be the event "The bead is a square."
To find $P(E \bigcup F)$, we need to count the beads which are green, or square, or both.

	△	□	Total
Red	1	8	9
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Let E be the event "The bead is green," and F be the event "The bead is a square."

To find $P(E \bigcup F)$, we need to count the beads which are green, or square, or both.

- The 64 **green** beads, even though they are triangular
-
-
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Let E be the event "The bead is green," and F be the event "The bead is a square."
To find $P(E \bigcup F)$, we need to count the beads which are green, or square, or both.

- The 64 **green** beads, even though they are triangular
- The 8 square beads, even though they are red
-
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Let E be the event "The bead is green," and F be the event "The bead is a square."
To find $P(E \bigcup F)$, we need to count the beads which are green, or square, or both.

- The 64 **green** beads, even though they are triangular
- The 8 square beads, even though they are red
- The 32 square beads, even though they are blue
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Let E be the event "The bead is green," and F be the event "The bead is a square."
To find $P(E \bigcup F)$, we need to count the beads which are green, or square, or both.

- The 64 **green** beads, even though they are triangular
- The 8 square beads, even though they are red
- The 32 square beads, even though they are blue
- The 16 **green** and *square* beads.

Numbers in a Table

	△	□	Total
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Blue	2	32	34
Total	67	56	123

Let E be the event "The bead is green," and F be the event "The bead is a square."
To find $P(E \bigcup F)$, we need to count the beads which are green, or square, or both.

$$64 + 16 + 8 + 32 = 120 \text{ beads in } E \bigcup F.$$

Hence

$$P(E \bigcup F) = \frac{120}{123}.$$

?(2.4) Numbers in a Table (And)

	$\triangle$	$\square$	Total
Red	1	8	9
Green	64	16	80
Blue	2	32	34
Total	67	56	123

Let E be the event "The bead is green," and F be the event "The bead is a square."
Find $P(E \bigcap F)$.

Numbers in a Table (And)

	$\triangle$	$\square$	Total
Red	1	8	9
Green	64	**16**	80
Blue	2	32	34
Total	67	56	123

Let E be the event "The bead is green," and F be the event "The bead is a square."
The event $E \bigcap F$ is "The bead is green **and** square:"

$$16 \text{ beads in } E \bigcap F.$$

Hence

$$P(E \bigcap F) = \boxed{\frac{16}{123}}.$$

Unions and Intersections Formula

The probability of event E or event F occurring (or both) is

$$P(E \bigcup F) = P(E) + P(F) - P(E \bigcap F).$$

This formula is useful for checking your answers, and if you do not know what outcomes are in the events.

Unions and Intersections Formula

	△	□	Total
Red	1	8	9
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Blue	2	32	34
Total	67	56	123

Let E be the event “The bead is green,” and F be the event “The bead is a square.”
The event $E \bigcup F$ is “The bead is green **or** square.” The event $E \bigcap F$ is “The bead is green **and** square.”

$$
\begin{aligned}
P(E) &= \frac{80}{123} \\
P(F) &= \frac{56}{123} \\
P(E \bigcap F) &= \frac{16}{123}
\end{aligned}
$$

Unions and Intersections Formula

$$\begin{aligned} P(E) &= \frac{80}{123} \\ P(F) &= \frac{56}{123} \\ P(E \bigcap F) &= \frac{16}{123} \end{aligned}$$

Thus

$$\begin{aligned} P(E \bigcup F) &= P(E) + P(F) - P(E \bigcap F) \\ &= \frac{80}{123} + \frac{56}{123} - \frac{16}{123} \\ &= \frac{80 + 56 - 16}{123} = \boxed{\frac{120}{123}} \end{aligned}$$

$$\begin{aligned} P(E \bigcup F) &= P(E) + P(F) - P(E \bigcap F) \\ &= \frac{80}{123} + \frac{56}{123} - \frac{16}{123} \\ &= \frac{80 + 56 - 16}{123} = \boxed{\frac{120}{123}} \end{aligned}$$

If we just add the number of green beads and the number of square beads,

$$80 + 56 = 136,$$

we get an answer that is too big. It double counts the 16 beads that are green and square, so that is why the formula subtracts the probability of the intersection. In the numerator, it subtracts the number of beads that are green and square:

$$136 - 16 = 120,$$

the correct numerator and the actual number of beads that are green and square.

End

You have homework due tonight.