Intro to Contemporary Math

Continuous Probability Introduction

Department of Mathematics

UK

Announcement

- You have a new homework assignment. It is due next Monday.
- Mini-exam 2 is next Wednesday.

Notice for today's lecture:
Slides with gray shading are for watching.
Slides with colored highlighting have definitions and examples.

Discrete Probability

At this point, we have been doing probability on discrete sets: sets of separate, individual objects like marbles, people, or whole numbers.
For example, if we have these 7 marbles:

then the probability of drawing a blue marble is

$$\frac{4 \text{ blue marbles}}{7 \text{ marbles total}} = \frac{4}{7},$$

At this point, we have been doing probability on discrete sets: sets of separate, individual objects like marbles, people, or whole numbers.
For example, if we have these 7 marbles:

then the probability of drawing a blue marble is

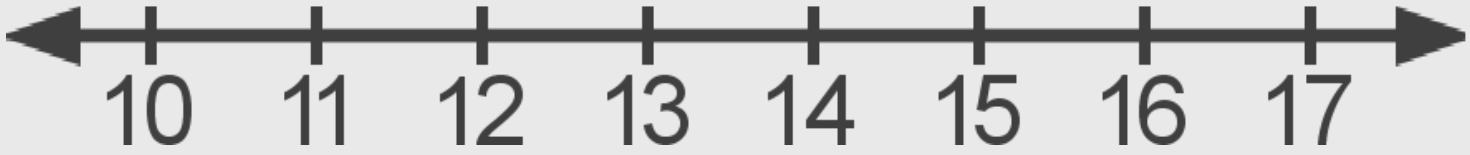
$$\frac{4 \text{ blue marbles}}{7 \text{ marbles total}} = \frac{4}{7},$$

the **ratio** of the *amount* of outcomes (marbles) in the event "we draw a blue marble" over the *total amount* of outcomes. The blue marbles take up 4/7ths of the total amount of marbles.

Continuous Probability

Many measurements like height and mass involve real numbers: not just whole numbers, but fractions, roots, and everything in between.

Picking a Random Real Number

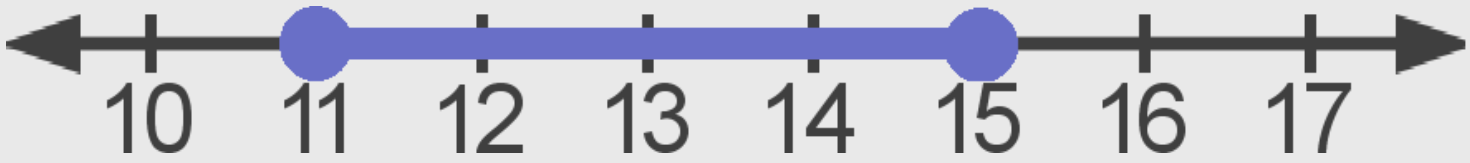


Pick a number between 10 and 17.

- It could be a whole number like 13, or
- a rational number (fraction) like 14.5, or
- an irrational number like $\sqrt{111} \approx 10.54$.

There are **infinitely many choices (outcomes)** between 10 and 17.

Picking a Random Real Number

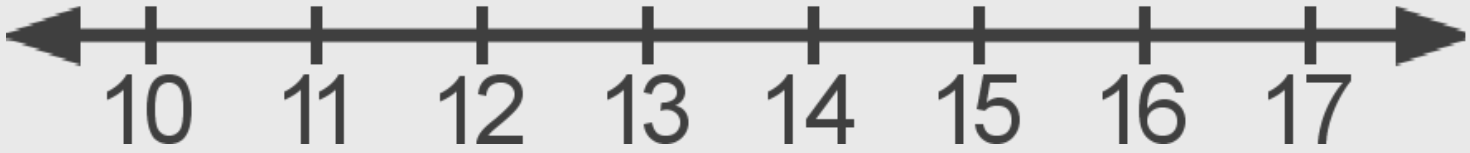


If we pick a number between 10 and 17, what is the probability that it is between 11 and 15?

- Some outcomes (numbers) in the event are 11, 12, 13.5.
- Some outcomes not in the event are 10.5 (too small), 16 (too big).

There are **infinitely many choices (outcomes)** between 11 and 15.

Picking a Random Real Number



This problem boils down to picking a random real number between 10 and 17 and finding the probability of picking a number between 11 and 15. Unlike earlier problems, our sample space, the set of real numbers between 10 and 17, not only has an infinite amount of outcomes, but these outcomes lie in **a continuous range**, a line, instead of being separate objects.

Intervals

Let c and d be any real numbers with c less than (left of) d.

- An *interval* between c and d is a set of *all real numbers between c and d*.
  A number is inside an interval if it is between c and d.
-
-
-
-
-

Intervals

Let c and d be any real numbers with c less than (left of) d.

- An *interval* between c and d is a set of *all real numbers between c and d*.
  A number is inside an interval if it is between c and d.
- The numbers c and d are the *endpoints* of the interval.
  They may or may not be included in the interval.
- $[c, d]$: include c and d
-
-
-

Intervals

Let c and d be any real numbers with c less than (left of) d.

- An *interval* between c and d is a set of *all real numbers between c and d*.
  A number is inside an interval if it is between c and d.
- The numbers c and d are the *endpoints* of the interval.
  They may or may not be included in the interval.
- $[c, d]$: include c and d
- $[c, d)$: include c only
- $(c, d]$: include d only
- (c, d): don't include endpoints

Drawing Intervals

Use **dots** to indicate endpoints:

- Use a **closed dot** to **include** the endpoint.
- Use an **open dot** to **not include** the endpoint.

Drawing Intervals

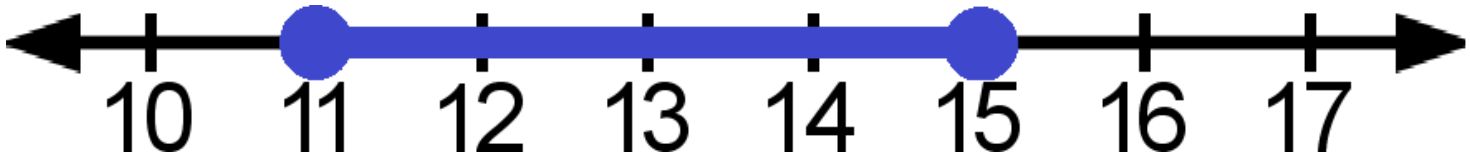
Use **dots** to indicate endpoints:

 Use a **closed dot** to **include** the endpoint.

 Use an **open dot** to **not include** the endpoint.

 Then draw a **line between dots** to include the **numbers between the endpoints**.

Interval Example



This interval has endpoints 11 and 15.
It includes 11, and 15, so it is written as

$$[11, 15].$$

There are infinitely many real numbers in this interval.

Picking a Random Real Number

When picking a random real number, our sample spaces and events will be intervals.

-
-

Picking a Random Real Number

When picking a random real number, our sample spaces and events will be intervals.

- Back in discrete probability, we *found the sizes of our events and sample spaces* by *counting* the amount of *outcomes* in them.
-

Picking a Random Real Number

When picking a random real number, our sample spaces and events will be intervals.

- Back in discrete probability, we *found the sizes of our events and sample spaces* by *counting* the amount of *outcomes* in them.
- Since we cannot count every real number in an interval, we need a way of measuring the size of an interval.

Length of an Interval

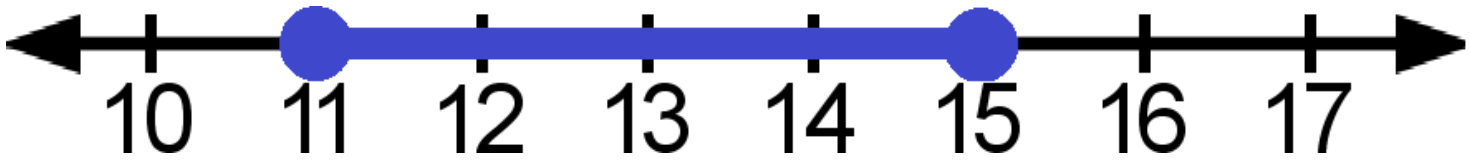
The **length** of an interval with endpoints c and d, $c \leq d$, is

$$d - c.$$

Right Endpoint minus **Left Endpoint**

The length should be a nonnegative number.

Length Example



The interval $[11, 15]$ has length

$$15 - 11 = \boxed{4}.$$

The interval is four units long.

?(3.1) Length Practice

Find the length of the following interval:

Length Practice

Find the length of the following interval:

Right endpoint is 16 and left endpoint is 13, so the length is

$$16 - 13 = \boxed{3}$$

?(3.2) Length Practice

Find the length of the following interval:
$[62, 104]$

Length Practice

Find the length of the following interval:
$[62, 104]$
Right endpoint is 104 and left endpoint is 62, so the length is

$$104 - 62 = \boxed{42}$$

Probability with Intervals

- When picking a random real number, our sample spaces and events will be intervals.
-

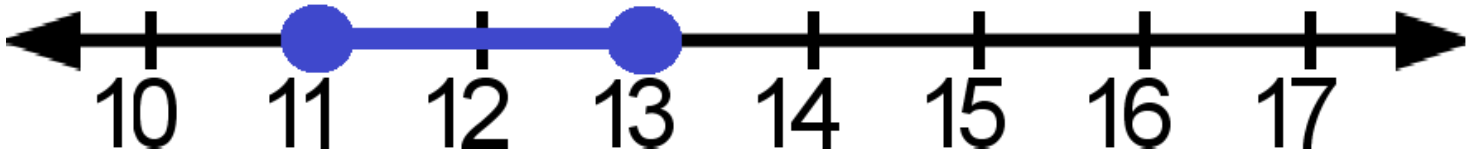
Probability with Intervals

- When picking a random real number, our sample spaces and events will be intervals.
- The probability of any event is

$$\frac{\text{Length of the event interval}}{\text{Length of the sample space interval}}$$

Throughout the rest of this chapter, all intervals will include endpoints.

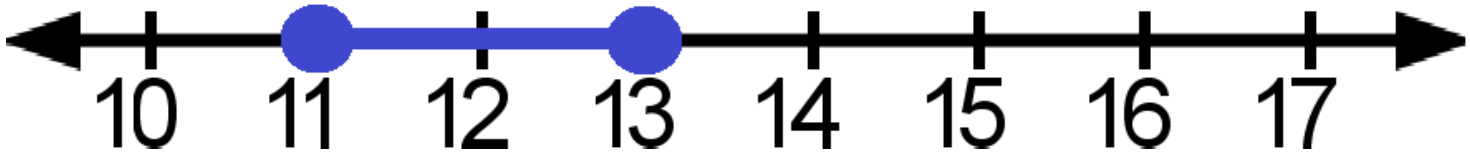
Probability of an Event Interval



We are picking a random real number between 10 and 17. What is the probability that the chosen number is between 11 and 13?

-
-

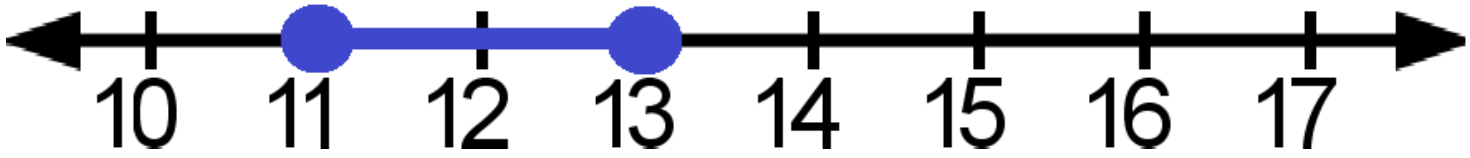
Probability of an Event Interval



We are picking a random real number between 10 and 17.
What is the probability that the chosen number is between 11 and 13?

- Sample space: the interval $[10, 17]$.
-

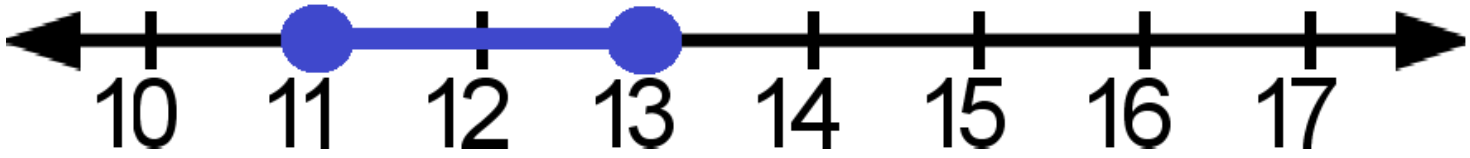
Probability of an Event Interval



We are picking a random real number between 10 and 17.
What is the probability that the chosen number is between 11 and 13?

- Sample space: the interval $[10,17]$. Length: $17-10=7$.
-

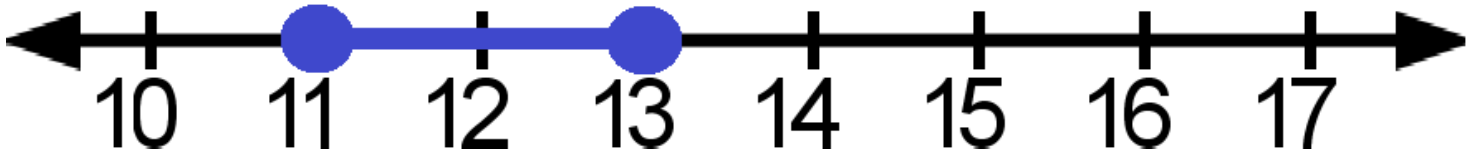
Probability of an Event Interval



We are picking a random real number between 10 and 17. What is the probability that the chosen number is between 11 and 13?

- Sample space: the interval $[10, 17]$. Length: $17 - 10 = 7$.
- Event: (we pick a real number inside) the interval $[11, 13]$.

Probability of an Event Interval

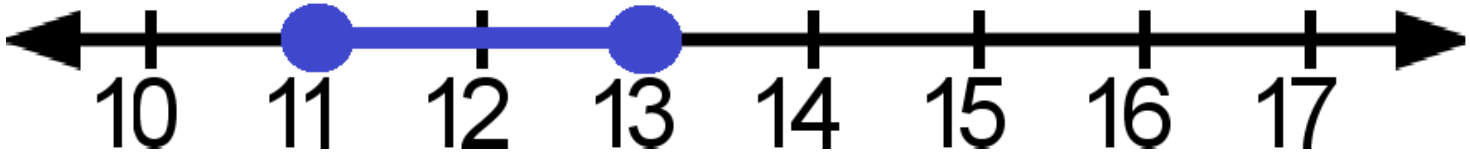


We are picking a random real number between 10 and 17. What is the probability that the chosen number is between 11 and 13?

- Sample space: the interval $[10, 17]$. Length: $17 - 10 = 7$.
- Event: (we pick a real number inside) the interval $[11, 13]$.

$$\textit{Length}: \ 13 - 11 = 2.$$

Probability of an Event Interval



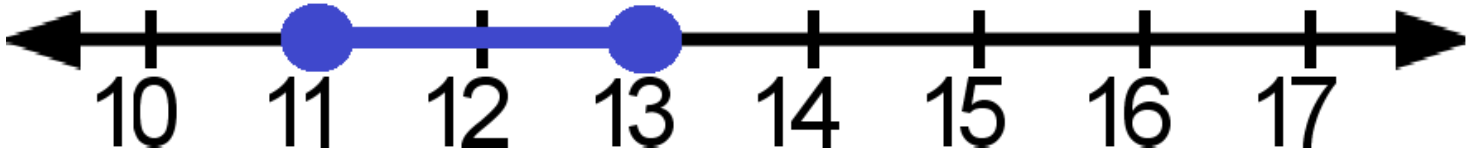
We are picking a random real number between 10 and 17. What is the probability that the chosen number is between 11 and 13?

- Sample space: the interval $[10, 17]$. Length: $17 - 10 = 7$.
- Event: (we pick a real number inside) the interval $[11, 13]$.

$$\textit{Length}: \ 13 - 11 = 2.$$

- Probability:

$$\frac{\text{Length of event}}{\text{Length of sample space}} = \frac{2}{7}.$$



We are picking a random real number between 10 and 17. What is the probability that the chosen number is between 11 and 13?

- Sample space: the interval $[10, 17]$. Length: $17 - 10 = 7$.
- Event: (we pick a real number inside) the interval $[11, 13]$.

$$\textit{Length}: \ 13 - 11 = 2.$$

- Probability:

$$\frac{\text{Length of event}}{\text{Length of sample space}} = \frac{2}{7}.$$

Notice that the event interval takes up 2/7ths of the total length of the sample space.

?(3.3) Probability of an Event Interval

We are picking a random real number between 23 and 91. What is the probability that the chosen number is between 34 and 47?

Hints:

- Find the length of the sample space.
- Find the length of the event interval.
- Divide appropriately.

Probability of an Event Interval

We are picking a random real number between 23 and 91. What is the probability that the chosen number is between 34 and 47?

- Sample space: the interval $[23, 91]$.

$$\textit{Length}: \ 91 - 23 = 68.$$

- Event: (we pick a real number inside) the interval $[34, 47]$.

$$\textit{Length}: \ 47 - 34 = 13.$$

- Probability:

$$\frac{\text{Length of event}}{\text{Length of sample space}} = \frac{13}{68}.$$

Comparing Discrete and Continuous Probability

- Discrete probability: used when the sample space is a set of separate, individual objects
- Continuous probability: used when the sample space is an interval of real numbers (we are picking a random real number)

Comparing Discrete and Continuous Probability

Let Ω be a sample space and E be an event.

- Discrete probability: The probability of E is:

$$\text{Number of objects in } E$$

$$\text{divided by...}$$

$$\text{Number of objects in } \Omega$$

- Continuous probability: The probability of an interval event E is:

$$\text{Length of } E$$

$$\text{divided by...}$$

$$\text{Length of } \Omega$$

End

- You have a new homework assignment. It is due next Monday.
- Mini-exam 2 is next Wednesday.