# Intro to Contemporary Math

## Unions and Intersections of Intervals

Department of Mathematics

UK

# Announcement

- You have a homework assignment due next Monday.
- Mini-exam 2 is next Wednesday.

# Unions and Intersections of Intervals

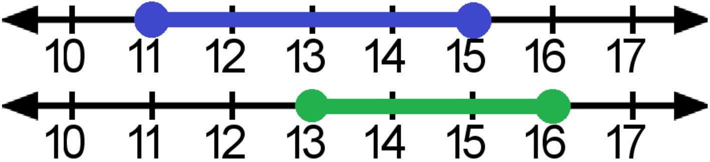
Interval events on the real line can be combined to form more complicated events using unions and intersections to represent outcomes in one event or the other (unions) or outcomes in the overlap of both events (intersections).

# Unions and Intersections of Intervals

Let $\Omega$ be an interval of real numbers, and let $E$ and $F$ be event intervals in $\Omega$.

- A real number outcome is in $E$ if it is between the endpoints of $E$.
- The event $E \bigcup F$ is the **union** of $E$ and $F$. It is the set of real number outcomes in $E$ **or** in $F$ (or in both).
  - The union could be a larger interval, or two separate intervals, depending on whether $E$ and $F$ overlap.
- The event $E \bigcap F$ is the **intersection** of $E$ and $F$. It is the set of real number outcomes in $E$, **and** in $F$.
  - If $E$ and $F$ overlap, then the intersection is the interval formed by the overlap.

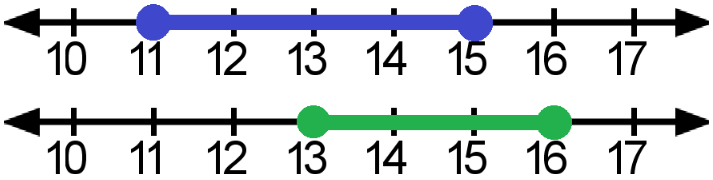
# Union of Two Overlapping Intervals



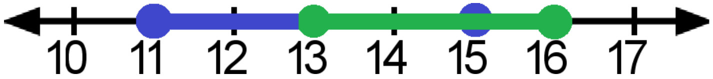

Let $\Omega = [10, 17]$, $E = [11, 15]$ and $F = [13, 16]$. The event $E \bigcup F$ consists of real numbers between 11 and 15, **or** between 13 and 16:

- The real number 12 is in $E \bigcup F$, because 12 is in $E$ (between 11 and 15).
- The real number 15.5 is in $E \bigcup F$, because 15.5 is in $F$ (between 13 and 16).
- The real number 14 is in $E \bigcup F$, because 14 is in $E$ and in $F$ (at least one of them).
- The real number 17 is not in $E \bigcup F$, because 17 is not in $E$ nor in $F$.

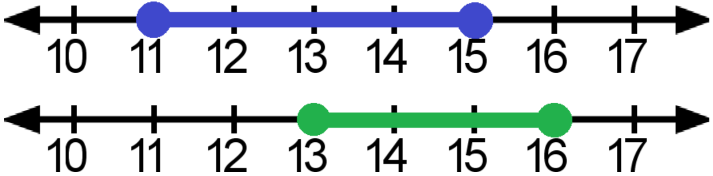
# Union of Two Overlapping Intervals




Let $\Omega = [10, 17]$, $E = [11, 15]$ and $F = [13, 16]$. The event $E \bigcup F$ consists of real numbers between 11 and 15, **or** between 13 and 16:



# Union of Two Overlapping Intervals




Let $\Omega = [10, 17]$, $E = [11, 15]$ and $F = [13, 16]$. The event $E \bigcup F$ consists of real numbers between 11 and 15, **or** between 13 and 16:

# Union of Two Overlapping Intervals

Let $\Omega = [10, 17]$, $E = [11, 15]$ and $F = [13, 16]$. The event $E \bigcup F$ consists of real numbers between 11 and 15, **or** between 13 and 16:

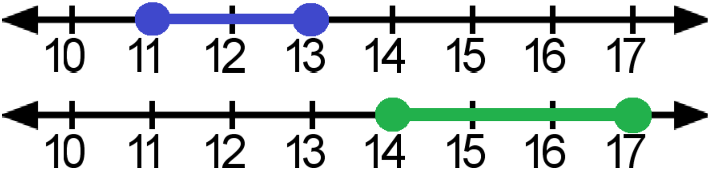

Thus, $E \bigcup F$ is actually the interval $[11, 16]$. Its length is

$$16 - 11 = 5.$$

Thus,

$$P(E \bigcup F) = \frac{\textit{Length of } E \bigcup F}{\textit{Length of } \Omega} = \frac{16 - 11}{17 - 10} = \boxed{\frac{5}{7}}$$

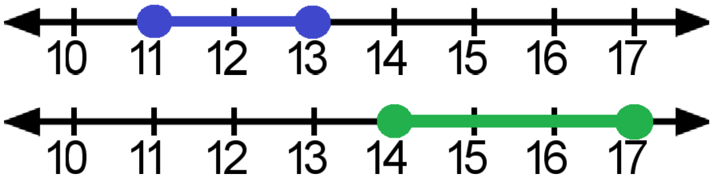
# Union of Two Separate Intervals




Let $\Omega = [10, 17]$, $E = [11, 13]$ and $F = [14, 17]$. The event $E \bigcup F$ consists of real numbers between 11 and 13, **or** between 14 and 17:

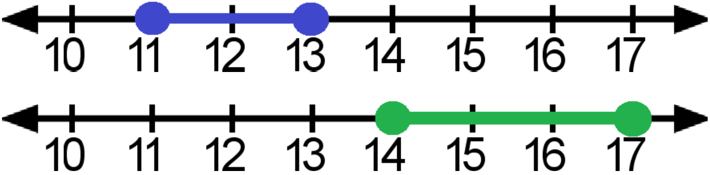
- The real number 12 is in $E \bigcup F$, because 12 is in $E$.
- The real number 15.5 is in $E \bigcup F$, because 15.5 is in $F$.
- The real number 13.5 is not in $E \bigcup F$, because 13.5 is not in $E$ nor in $F$.

# Union of Two Separate Intervals

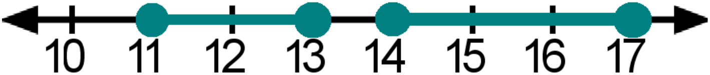



Let $\Omega = [10, 17]$, $E = [11, 13]$ and $F = [14, 17]$. The event $E \bigcup F$ consists of real numbers between 11 and 13, **or** between 14 and 17:

# Union of Two Separate Intervals

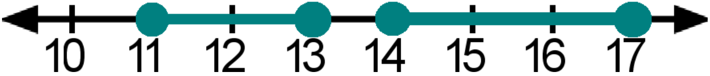



Let $\Omega = [10, 17]$, $E = [11, 13]$ and $F = [14, 17]$. The event $E \bigcup F$ consists of real numbers between 11 and 13, **or** between 14 and 17:



# Union of Two Separate Intervals

Let $\Omega = [10, 17]$, $E = [11, 13]$ and $F = [14, 17]$. The event $E \bigcup F$ consists of real numbers between 11 and 13, **or** between 14 and 17:




Thus, $E \bigcup F$ is actually two intervals. It has a total length found by adding (combining) the lengths of $E$ and $F$:

$$\text{Length of } E: \; 3 - 1 = 2$$

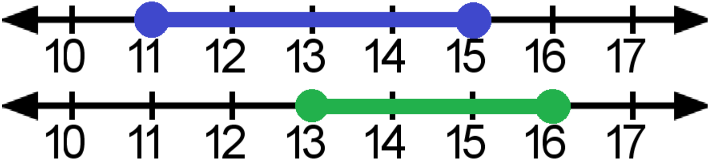
$$\text{Length of } F: \; 7 - 4 = 3$$

$$\text{Total length of } E \bigcup F: \; 2 + 3 = 5.$$

Hence

$$P(E \bigcup F) = \frac{\textit{Total length of } E \bigcup F}{\textit{Length of } \Omega} = \boxed{\frac{5}{7}}.$$

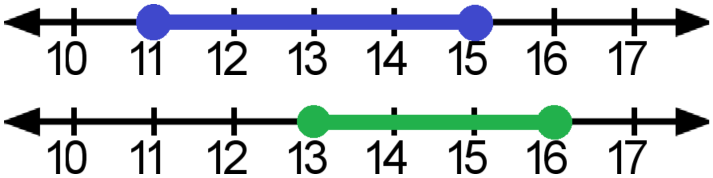
# Intersection of Two Intervals



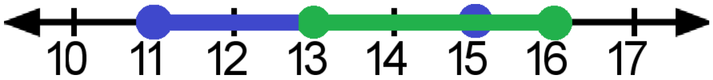

Let $\Omega = [10, 17]$, $E = [11, 15]$ and $F = [13, 16]$. The event $E \bigcap F$ consists of real numbers between 11 and 15, **and** between 13 and 16:

- The real number 12 is not in $E \bigcap F$, because 12 is in $E$, but not in $F$.
- The real number 15.5 is not in $E \bigcap F$, because 15.5 is in $F$, but not in $E$.
- The real number 14 is in $E \bigcap F$, because 14 is in $E$ and in $F$ (both of them).

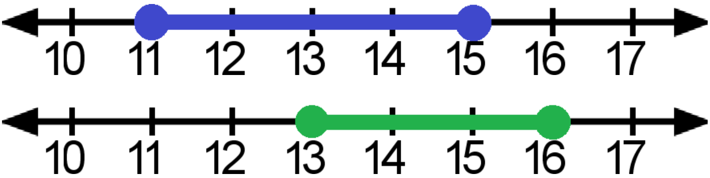
# Intersection of Two Intervals




Let $\Omega = [10, 17]$, $E = [11, 15]$ and $F = [13, 16]$. The event $E \bigcap F$ consists of real numbers between 11 and 15, **and** between 13 and 16:



# Intersection of Two Intervals




Let $\Omega = [10, 17]$, $E = [11, 15]$ and $F = [13, 16]$. The event $E \bigcap F$ consists of real numbers between 11 and 15, **and** between 13 and 16:

# Intersection of Two Intervals

Let $\Omega = [10, 17]$, $E = [11, 15]$ and $F = [13, 16]$. The event $E \bigcap F$ consists of real numbers between 11 and 15, **and** between 13 and 16:


Thus, $E \bigcap F$ is actually the interval $[13, 15]$. Its length is

$$5 - 3 = 2.$$

Hence

$$P(E \bigcap F) = \frac{Length\ of\ \ E \bigcap F}{Length\ of\ \ \Omega} = \boxed{\frac{2}{7}}.$$

# ?(4.1) Union/Intersection Practice 1

Let $\Omega$ be the interval $[8, 24]$, $E$ be the interval $[12, 16]$, and $F$ be the interval $[15, 19]$:


If we pick a random real number between 8 and 24, find the **probability** of the event $E \bigcup F$.

Hints:

1. Identify the event $[12, 16] \bigcup [15, 19]$ as an interval.
2. What is the length of the union?
3. What is the length of the sample space?
4. Find $P([12, 16] \bigcup [15, 19])$. Type and send a fraction.

# Union/Intersection Practice 1

Let $\Omega$ be the interval $[8, 24]$, $E$ be the interval $[12, 16]$, and $F$ be the interval $[15, 19]$:


If we pick a random real number between 8 and 24, find the **probability** of the event $E \bigcup F$.

Hints:

1. Identify the event $[12, 16] \bigcup [15, 19]$ as an interval.
2. What is the length of the union?
3. What is the length of the sample space?
4. Find $P([12, 16] \bigcup [15, 19])$. Type and send a fraction.

# Union/Intersection Practice 1

Let $\Omega$ be the interval $[8, 24]$, $E$ be the interval $[12, 16]$, and $F$ be the interval $[15, 19]$:


If we pick a random real number between 8 and 24, find the **probability** of the event $E \bigcup F$.

$$E \bigcup F = [12, 19].$$

$$P(E \bigcup F) = \frac{\text{Length of } E \bigcup F}{\text{Length of } \Omega} = \frac{19 - 12}{24 - 8} = \boxed{\frac{7}{16}}$$

# ?(4.2) Union/Intersection Practice 2

Let $\Omega$ be the interval $[24, 47]$, $E$ be the interval $[29, 34]$, and $F$ be the interval $[36, 43]$:

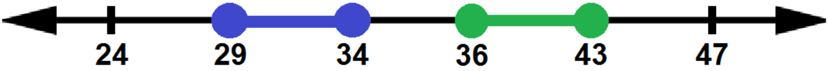

If we pick a random real number between 24 and 47, find the **probability** of the event $E \bigcup F$.

Hints:

1. Identify the event $[29, 34] \bigcup [36, 43]$ as a union of two separate intervals.
2. What is the total length of the union?
3. What is the length of the sample space?
4. Find $P([29, 34] \bigcup [36, 43])$. Type and send a fraction.

# Union/Intersection Practice 2

Let $\Omega$ be the interval $[24, 47]$, $E$ be the interval $[29, 34]$, and $F$ be the interval $[36, 43]$:



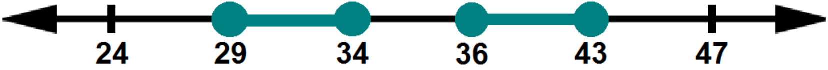

If we pick a random real number between 24 and 47, find the **probability** of the event $E \bigcup F$.

Hints:

1. Identify the event $[29, 34] \bigcup [36, 43]$ as a union of two separate intervals.
2. What is the total length of the union?
3. What is the length of the sample space?
4. Find $P([29, 34] \bigcup [36, 43])$. Type and send a fraction.

# Union/Intersection Practice 2

Let $\Omega$ be the interval $[24, 47]$, $E$ be the interval $[29, 34]$, and $F$ be the interval $[36, 43]$:




If we pick a random real number between 24 and 47, find the **probability** of the event $E \bigcup F$.

$$E \bigcup F = [29, 34] \bigcup [36, 43]$$

$$\begin{aligned} P(E \bigcup F) &= \frac{\text{Total Length of } E \bigcup F}{\text{Length of } \Omega} \\ &= \frac{(34 - 29) + (43 - 36)}{47 - 24} \\ &= \frac{5 + 7}{23} = \boxed{\frac{12}{23}} \end{aligned}$$

# ?(4.3) Union/Intersection Practice 3

Let $\Omega$ be the interval $[47, 78]$, $E$ be the interval $[51, 60]$, and $F$ be the interval $[54, 63]$:


If we pick a random real number between 47 and 78, find the **probability** of the event $E \bigcap F$.

Hints:

1. Identify the event $[51, 60] \bigcap [54, 63]$ as a smaller interval.
2. What is the length of the intersection?
3. What is the length of the sample space?
4. Find $P([51, 60] \bigcap [54, 63])$. Type and send a fraction.

# Union/Intersection Practice 3

Let $\Omega$ be the interval $[47, 78]$, $E$ be the interval $[51, 60]$, and $F$ be the interval $[54, 63]$:


If we pick a random real number between 47 and 78, find the **probability** of the event $E \bigcap F$.

Hints:

1. Identify the event $[51, 60] \bigcap [54, 63]$ as a smaller interval.
2. What is the length of the intersection?
3. What is the length of the sample space?
4. Find $P([51, 60] \bigcap [54, 63])$. Type and send a fraction.

# Union/Intersection Practice 3

Let $\Omega$ be the interval $[47, 78]$, $E$ be the interval $[51, 60]$, and $F$ be the interval $[54, 63]$:


If we pick a random real number between 47 and 78, find the **probability** of the event $E \bigcap F$.

$$E \bigcap F = [54, 60].$$

$$P(E \bigcap F) = \frac{\text{Length of } E \bigcap F}{\text{Length of } \Omega} = \frac{60 - 54}{78 - 47} = \boxed{\frac{6}{31}}$$

# End

- You have a homework assignment due next Monday.
- Mini-exam 2 is next Wednesday.