Worksheet #4
August 31, 2018
4 Points

~~Circle one name.~~

Name: Solutions Name: _____ Name: _____

1. Below is a preference schedule giving the voter preferences in an election with four candidates, A, B, C, and D.

Number of Voters	12	2	7	6	9
					$= 36$
1st choice	A	C	D	B	C
2nd choice	B	D	C	D	B
3rd choice	C	B	B	C	D
4th choice	D	A	A	A	A

(a) Is there a Majority Candidate? If so, who? If not, why not?

$\frac{36}{2} = 18 \rightarrow 19$ needed for majority. No candidate has 19 or more votes.

(b) Which Candidate wins if the Pairwise Comparison Method is used? Show all the pairs, and give the pairwise points.

A vs. B	A vs. C	A vs. D	B vs. C	B vs. D	C vs. D
12 | 2, 7, 6, 9	12 | 2, 7, 6, 9	12 | 2, 7, 6, 9	12, 6 | 2, 7, 9	12, 6, 9 | 2, 7	12, 2, 9 | 7, 6
12 | 24	12 | 24	12 | 24	18 | 18	27 | 9	23 | 13
B wins	C wins	D wins	B Ties C	B wins	C wins

Points
A: $\rightarrow 0$
B: 1, .5, 1 $\rightarrow 2.5$
C: 1, .5, 1 $\rightarrow 2.5$
D: 1 $\rightarrow 1$

B and C tie, so no clear winner.

(c) Is there a candidate who against each of the other three in pairwise comparisons (such a candidate is called a Condorcet candidate)? If so, who? If not, why not? How many pairwise points are required to be a Condorcet candidate in this election?

No Condorcet candidate; no candidate won all matchups. 3 points are needed to be a Condorcet candidate.

2. Below is a preference schedule giving the voter preferences in an election with four candidates, A, B, C, and D.

Number of Voters	13	12	6	3
1st choice	B	D	A	D
2nd choice	C	A	C	C
3rd choice	A	C	B	B
4th choice	D	B	D	A

13 + 12 + 6 + 3 = 34

(a) Which candidate wins if the Plurality Method is used? How do you know? Is there a Plurality Candidate? How do you know? Also, is there a Majority Candidate? How do you know?

D wins using Plurality because D has the most first place votes with 15. This also makes D a Plurality candidate. (A Plurality candidate is the candidate that wins using Plurality).

$\frac{34}{2} = 17 + 1 = 18$

There is no majority candidate. 18 votes are needed to be a majority candidate and no candidate received more than 15.

(b) Which candidate wins if the Borda Count Method is used? How many Borda points does each candidate receive?

A: $13(2) + 12(3) + 6(4) + 3(1) = 89$

B: $13(4) + 12(1) + 6(2) + 3(2) = 82$

C: $13(3) + 12(2) + 6(3) + 3(3) = 90$

D: $13(1) + 12(4) + 6(1) + 3(4) = 79$

C wins using Borda

(c) Which candidate wins if the Plurality With Elimination Method is used? Show your work clearly.

13	12	6	3
B	D	A	D
~~C~~	A	~~C~~	~~C~~
A	~~C~~	B	B
D	B	D	A

C has fewest 1st place votes. Eliminate C. →

13	12	6	3
B	D	~~A~~	D
~~A~~	~~A~~	B	~~B~~
D	B	D	A

A has fewest 1st place votes. Eliminate A. →

(13)	12	(6)	3
B	D	B	D
D	B	D	B

B has >18 votes, majority
B wins

B wins using PwE

(d) Which candidate wins if the Pairwise Comparison Method is used? Show all the pairs, and give the pairwise points. Is there a Condorcet Candidate? How do you know?

A vs. B		A vs. C		A vs. D		B vs. C		B vs. D		C vs. D	
12	13	12	13	13	12	13	12	13	12	13	12
6	3	6	3	6	3		6	6	3	6	3
							3				
18	16	18	16	19	15	13	21	19	15	19	15
A wins		A wins		A wins		C wins		B wins		C wins	

Points
(A: 3)
B: 1
C: 2
D: 0

A wins using Pairwise Comparison

3. An election takes place with 50 voters and 5 candidates.

(a) How many different preference ballots are possible? Explain your answer.

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

5 1st choices, 4 2nd choices, etc.

(b) What is the sum of all of the Borda points of all of the candidates? Explain your answer.

50 voters · 5 pts. for 1st place = 250

$50 \cdot 4 = 200$

$50 \cdot 3 = 150$

$50 \cdot 2 = 100$

$50 \cdot 1 = +50$

750 pts.

(c) How many pairwise comparisons are made for the Pairwise Comparison Method? Explain your answer.

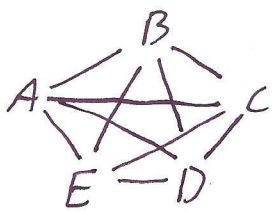
A vs. B, A vs. C, A vs. D, A vs. E } 4 A's

B vs. C, B vs. D, B vs. E } 3 more Bs

C vs. D, C vs. E } 2 more Cs

D vs. E } 1 last matchup

$4 + 3 + 2 + 1 = 10$ matchups



10 lines = 10 matchups