Graph Theory Worksheet #1
November 26, 2018
2 Points

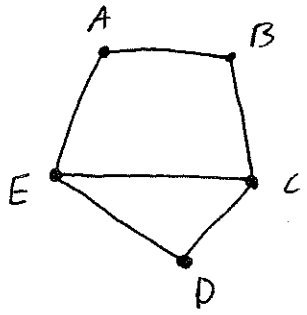
Circle one name.

Name: Solutions **Name:** _____ **Name:** _____

1. A certain graph G has vertex set $\{A, B, C, D, E\}$ and edge set

$$\{\{A, B\}, \{A, E\}, \{B, C\}, \{C, D\}, \{C, E\}, \{D, E\}\}.$$

(a) Sketch the graph in such a way that its edges do not cross, thus confirming that it is planar.



(b) What is the order of the graph?

5

(c) What is v?

5

(d) What is e?

6

(e) What is f?

3

(f) List the vertex degree sequence in increasing order.

2, 2, 2, 3, 3

(g) List the faces of the graph.

CDE, ABCE, ABCDE

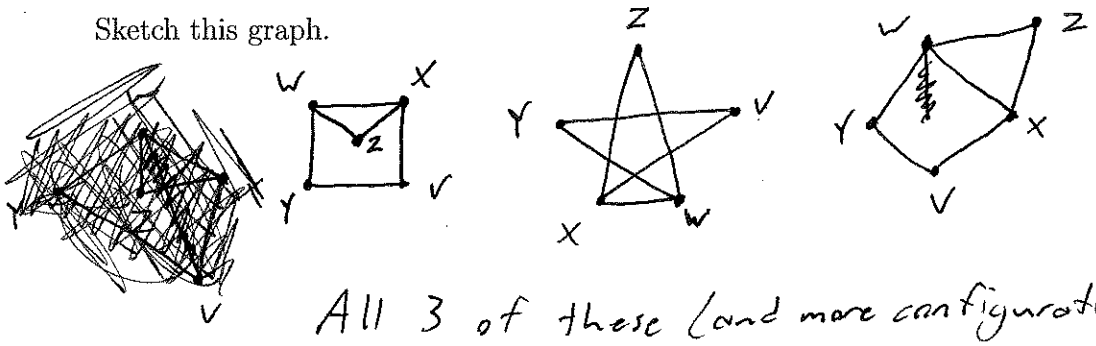
(h) Give the degree of each face.

CDE: 3 ABCE: 4 ABCDE: 5

(i) Another certain graph H has vertex set $\{V, W, X, Y, Z\}$ and edge set

$$\{\{V,X\},\{V,Y\},\{W,X\},\{W,Y\},\{W,Z\},\{X,Z\}\}.$$

Sketch this graph.



All 3 of these (and more configurations) are possible graphs.

Explain why the graphs G and H are isomorphic and give the correspondence between the vertices.

G and H are isomorphic because they can be rearranged to look identical to each other.

	1	2
• $A \longrightarrow$	V	Y
• $B \longrightarrow$	Y	V
• $C \longrightarrow$	W	X
• $D \longrightarrow$	Z	Z
• $E \longrightarrow$	X	W

2. A certain planar graph has vertex degree sequence 3, 3, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4.

→ CONNECTED

(a) How many vertices does it have?

14

(b) How many edges does it have?

sum of vertex degrees $= 2 \cdot e \rightarrow 48 = 2 \cdot e \rightarrow \frac{48}{2} = e \rightarrow \underline{24 = e}$

(c) How many faces does it have?

$V - e + f = 2 \rightarrow 14 - 24 + f = 2 \rightarrow f - 10 = 2 \rightarrow \underline{f = 12}$

(d) What is the sum of the face degrees?

sum of face degrees $= 2 \cdot e = 2 \cdot 24 = \underline{48}$