Probability Worksheet #6
October 5, 2018
2 Points

Circle one name.

Name: Solutions **Name:** _____ **Name:** _____

Note: $P(E|F) = \frac{\text{size of } E \cap F}{\text{size of } F}$.

Note: Two events E and F are said to be independent if $P(F|E) = P(F)$.

1. A deck of 10 cards has 2 suits (A–B) and 5 ranks (1–5).

Suit \ Rank	1	2	3	4	5
A	A1	A2	A3	A4	A5
B	B1	B2	B3	B4	B5

Let E be the event "A card with suit B is drawn."

Let F be the event "A card with rank 2 is drawn."

(a) $P(E) = ?$ $\frac{5}{10} = \frac{1}{2}$

(b) $P(F) = ?$ $\frac{2}{10} = \frac{1}{5}$

(c) $P(E \cap F) = ?$ $\frac{1}{10}$ (card B2)

(d) $P(E \cup F) = ?$ $\frac{6}{10}$ (all Bs as well as A2)

(e) $P(E|F) = ?$ $\frac{1}{2}$ ← one rank 2 card is in B; ← two rank 2 cards

(f) $P(F|E) = ?$ $\frac{1}{5}$ ← B2; ← five suit B cards

(g) Are the events E and F independent in this example? How do you know?

Yes, $P(E) = P(E|F)$ and $P(F) = P(F|E)$

(h) Does $P(E) \cdot P(F) = P(E \cap F)$ in this example?

$\frac{5}{10} \cdot \frac{2}{10} = \frac{10}{100} = \frac{1}{10}$ $P(E \cap F) = \frac{1}{10}$ $\frac{1}{10} = \frac{1}{10}$

$P(E) \cdot P(F)$

Yes

2. A deck of 8 cards has 2 suits and 5 ranks, but cards B4 and B5 are not included:

Suit \ Rank	1	2	3	4	5
A	A1	A2	A3	A4	A5
B	B1	B2	B3		

Let E be the event "A card with suit B is drawn." Let F be the event "A card with rank 2 is drawn."

(a) $P(E) = ?$ $\frac{3}{8}$

(b) $P(F) = ?$ $\frac{2}{8} = \frac{1}{4}$

(c) $P(E \cap F) = ?$ $\frac{1}{8}$ (B2)

(d) $P(E \cup F) = ?$ $\frac{4}{8}$ (all 3 Bs as well as A2)

(e) $P(E|F) = ?$ $\frac{1 \leftarrow B2}{2 \leftarrow \text{two rank 2 cards}}$

(f) $P(F|E) = ?$ $\frac{1 \leftarrow B2}{3 \leftarrow \text{three B cards}}$

(g) Are the events E and F independent in this example? How do you know?

No, $P(E) \neq P(E|F)$ and $P(F) \neq P(F|E)$

(h) Does $P(E) \cdot P(F) = P(E \cap F)$ in this example?

$P(E) \cdot P(F) = \frac{3}{8} \cdot \frac{1}{4} = \frac{3}{32}$ $\quad P(E \cap F) = \frac{1}{8}$ $\quad \frac{3}{32} \neq \frac{1}{8}$, NO

3. In a standard deck of 52 cards there are four suits and 13 ranks.

(a) Suppose a card is drawn from the deck at random, and it turns out to be the ace of spades. Suppose the ace of spades is returned to the deck ("with replacement"), and again a card is drawn at random. What is the probability that it is a spade?

$\frac{13 \text{ spades}}{52 \text{ cards total}} = \frac{1}{4}$

(b) Suppose a card is drawn from the deck at random, and it turns out to be the ace of spades. Suppose the ace of spades is NOT returned to the deck ("without replacement"), and a second card is drawn at random. What is the probability that it is a spade?

$\frac{12 \text{ spades}}{51 \text{ cards total}}$ since the Ace of Spades is gone, there is one fewer spade ($13-1=12$) and one fewer total cards than before ($52-1=51$).