Probability Worksheet #9
October 12, 2018
2 Points

Circle one name.

Name: Solutions **Name:** _____ **Name:** _____

Note: $P(E|F) = \frac{\text{size of } E \cap F}{\text{size of } F}$.

1. A jar contains 10 red marbles and 15 green marbles. Two marbles are selected from the jar, one at a time, without replacement.

   (a) What is the probability that the second marble is red given that the first marble is red?

   $\frac{9}{24} \rightarrow$ Original $\frac{10}{25}$ (−1 red) → New $\frac{9}{24}$ (−1 red)

   (b) What is the probability that the second marble is red, given that the first marble is green?

   $\frac{10}{24} \rightarrow$ Original $\frac{10}{24}$ (no change) → New $\frac{10}{24}$ (−1 green)

2. Suppose our sample space is the interval $\Omega = [0, 100]$. A point is selected at random from the sample space, with all points being equally likely.
   Let E be the event that the point is in the interval $[0, 50]$.
   Let F the event that the point is in the interval $[20, 80]$.
   Let G the event that the point is in the interval $[30, 90]$.

   (a) Are the events E and F independent? Show your calculations.

   $P(F) = \frac{60}{100} = \frac{3}{5}$ $P(E) = \frac{50}{100} = \frac{1}{2}$
   $P(F|E) = \frac{30}{50} = \frac{3}{5}$ $P(E|F) = \frac{30}{60} = \frac{1}{2}$
   $P(E) = P(E|F)$ → independent
   $P(F) = P(F|E)$
   Either of these calculations is sufficient – you don't need to do both.

   (b) Are the events E and G independent? Show your calculations.

   $P(E) = \frac{50}{100} = \frac{1}{2}$ $P(G) = \frac{60}{100} = \frac{3}{5}$
   $P(E|G) = \frac{20}{60} = \frac{1}{3}$ $P(G|E) = \frac{20}{50} = \frac{2}{5}$
   $P(E) \neq P(E|G)$ → not independent
   $P(G) \neq P(G|E)$

3. A jar contains beads which are either blue or white, and either round or pointy. There are 360 beads total, with 160 of them being pointy.

	Round	Pointy	Total
Blue	180	40	220
White	20	120	140
Total	200	160	360

ADD across

360 − 160

Suppose that:
P(Bead is White|Bead is Round) = 10%, and
P(Bead is Blue|Bead is Pointy) = 25%.
Fill in the table.

$P(\text{White} \mid \text{Round}) = 10\% = \frac{10}{100} = \frac{1}{10}$

$\frac{10}{100} = \frac{x}{200}$

$2{,}000 = 100x$

$x = 20$ white round beads

$200 - 20 = 180$ blue round beads

$P(\text{blue} \mid \text{pointy}) = 25\% = \frac{25}{100} = \frac{1}{4}$

$\frac{25}{100} = \frac{x}{160} = \frac{1}{4}$

$4x = 160$

$x = 40$ blue pointy beads

$160 - 40 = 120$ white pointy beads