Fair — Equitable — Envy-Free

1 Fair

1.1 What is Fair?

A compensation is **fair** if each person feels they have received greater than or equal to their fair share.

1.2 How Can We Accomplish This?

1. Assume there is only one item.

2. Calculate the average bid m.

3. Select as the winner someone whose bid is greater than or equal to the average bid m.

4. The winner pays each loser at least the loser's share, but must be careful not to pay too much and fall below their own fair share.

5. In the special case when $N = 2$, and two people bid with $b_1 \leq b_2$, then the arrangement is fair only if the higher bidder wins and pays Person 1 x_1, where $b_1/2 \leq x_1 \leq b_2/2$. This is the **fairness interval**.

6. Note: Knaster's procedure also produces a compensation that is fair, since each person receives the same amount of surplus over their fair share.

2 Equitable

2.1 What is Equitable?

A compensation is **equitable** if each person ends up with the same XB ratio. This means that each person gets the same percent of their total bid.

2.2 How Can We Accomplish This?

1. Assume there is only one item and only two people.

2. When two people bid with $b_1 \leq b_2$, then the arrangement is equitable if the higher bidder Person 2 wins and pays Person 1

$$x_1 = \frac{(b_1 \times b_2)}{(b_1 + b_2)}.$$

3. Note: The Adjusted Winner procedure also produces a compensation that is equitable.

3 Envy-Free

3.1 What is Envy-Free?

A person will envy another person if he or she thinks the other person's compensation is higher than his or her own. An arrangement is **envy-free** if no one envies anyone.

3.2 How Can We Accomplish This?

1. Assume there is only one item.

2. Let h be the highest bid and b_2 be the second-highest bid.

3. Make the highest bidder win.

4. Make the winner pay each loser the same amount x_{Loser} between the second-highest and highest bidders' fair shares:

$$\frac{b_2}{N} \leq x_{\text{Loser}} \leq \frac{h}{N}.$$

This is the envy-free payment range.