Algebra Review

These problems and (and their solutions) are found in your textbook in the Algebra Review Appendix. You should, of course, try to work all of these problems without looking at the answers. Then check your answers. Repeat this process until you can solve every problem quickly and correctly. If you need more practice, try some of the other problems given at the end of A1 and A2. If you have any questions, see me, because *during this course I am expecting that you will all be adept at solving algebraic problems.*

1. The Real Numbers.
   (a) Define the following terms: integer, rational number, irrational number, real number.
   (b) Determine whether each of the following numbers is an integer, rational number, irrational number, or real number (give all correct answers).
      i. 1.
      ii. 875.
      iii. -83.
      iv. 0.
      v. $2/3$.
      vi. 8.71.
      vii. $\sqrt{2}$.
      viii. $-6\frac{1}{2}$.
      ix. π.
   (c) Convert to decimal form:
      i. $1/3$.
      ii. $5/8$.
   (d) Convert to fractional form:
      i. $1.181818\ldots$.
      ii. 8.71.
2. Inequalities and intervals.
   (a) Define the following terms: interval, closed interval, open interval, half-open interval, infinite interval.

(b) Graph each of the following intervals on a number line.

i. $x \leq 3$.

ii. $x > -2$.

iii. $-2 < x \leq 3$.

iv. $x < -1$.

v. $-1 \leq x \leq 2$.

vi. $x > 2$.

3. Absolute value.

(a) Determine $|-3|$, $|3|$, $|0|$, and $|5-9|$.

(b) How can absolute value be used to find the distance between two numbers on the number line?

(c) Find the distance between -2 and 3.

(d) Find the interval consisting of all real numbers x such that $|x-1| \leq 3$.

4. Exponents.

(a) Explain the meaning of each of the following:

i. a^n where n is a positive integer.

ii. $a^{n/m}$ where $a \geq 0$ and n and m are positive integers.

iii. a^{-x}.

iv. a^0 where $a \neq 0$.

(b) Evaluate the following (without using a calculator):

i. $9^{1/2}$.

ii. $27^{2/3}$.

iii. $8^{-1/3}$.

iv. $\left(\frac{1}{100}\right)^{-3/2}$.

v. 5^0.

(c) Describe the laws of exponents:

i. $a^r a^s = ?$

ii. $\frac{a^r}{a^s} = ?$

iii. $(a^r)^s = ?$

(d) Evaluate the following (without using a calculator):

i. $(2^{-2})^3$.

ii. $\dfrac{3^3}{3^{1/3}(3^{2/3})}$.

iii. $2^{7/4}(8^{-1/4})$.

(e) Solve each of the following equations for a:

i. $\dfrac{a^5}{a^2} = a^n$.

ii. $(a^n)^5 = a^{20}$.

5. Factoring.

(a) Factor the expression $3x^4 - 6x^3$.

(b) Simplify the expression $10(1-x)^4(x+1)^4 + 8(x+1)^5(1-x)^3$.

(c) Factor the expression $\frac{2}{3}x^{-1/3}(x+1) + x^{2/3}$.

(d) Simplify the quotient

$$\frac{4(x+3)^4(x-2)^2 - 6(x+3)^3(x-2)^3}{(x+3)(x-2)^3}.$$

6. Summation Notation.

(a) Use summation notation to express the following sums:

i. $1 + 4 + 9 + 16 + 25 + 36 + 49 + 64$.

ii. $(1-x_1)^2\Delta x + (1-x_2)^2\Delta x + \cdots + (1-x_{15})^2\Delta x$.

(b) Evaluate the following sums:

i.

$$\sum_{j=1}^{4}(j^2+1).$$

ii.

$$\sum_{j=1}^{3}(-2)^j.$$

7. Factoring polynomials and solving polynomial equations.

   (a) What is a polynomial?

   (b) What is the quadratic equation?

   (c) Factor the following polynomials using integer coefficients:

      i. $x^2 - 2x - 3$.

      ii. $2x^2 + x - 10$.

      iii. $x^3 - 8$.

      iv. $x^5 - 4x^3$.

   (d) Solve the following equations:

      i. $x^2 - 3x = 10$.

      ii. $1 - \frac{1}{x} - \frac{2}{x^2} = 0$.

      iii. $x^2 + 3x + 1 = 0$.

      iv. $x^2 + 18x + 81 = 0$.

      v. $x^2 + x + 1 = 0$.

8. Systems of equations.

   (a) Solve the system

$$\begin{aligned} 4x + 3y &= 13 \\ 3x + 2y &= 7 \end{aligned}$$

   (b) Solve the system

$$\begin{aligned} 2y^2 - x^2 &= 14 \\ x - y &= 1 \end{aligned}$$