

Shapes and Designs

Extensions 4

1. Prove that any simple, closed polygonal region can be dissected into triangles, even if the region is not convex.
2. When you have a dissection of a polygonal region into triangles, an *ear* is a triangle having two sides that are sides of the polygonal region. Prove that every dissection of a polygonal region has at least two ears.
3. Prove that any dissection into triangles of a polygonal region with V vertices must have exactly $V - 2$ triangles.
4. Let S be the sum of the interior angles of any polygonal region with V vertices (and hence V edges). Prove that $S = 180V - 360$.
5. Let S be the sum of the interior angles of all of the polygons in any polyhedron with V vertices, E edges, and F faces. Conjecture a formula for S .
6. Use Geometer's Sketchpad to draw a regular nonagon with radius length 1 inch. With side length 1 inch.
7. Use Maple to draw a regular nonagon with radius length 1 inch. With side length 1 inch.
8. I know of at least seven buildings in the world that are built in the shape of nonagons. Find a photograph of at least one of them.
9. Construct models to show that you can tile three-dimensional space (face-to-face) with
 - (a) Cubes.
 - (b) A mixture of regular tetrahedra and octahedra.
 - (c) Rhombic dodecahedra.
 - (d) Truncated octahedra (polyhedra with the space cluster $(4, 6, 6)$ at each vertex).
 - (e) Three-dimensional Swiss crosses, each consisting of six cubes glued onto the faces of a central seventh cube.