Quotes and Anecdotes

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1 Some of My Informal Thoughts about Teaching and Learning Mathematics

1. Don't neglect history. Mathematics is far, far from static. Everything you have studied was invented, discovered, or developed at some point in the past, and sometimes not so long ago! Many have the very strange impression that most of mathematics was finished centuries ago and is now fixed and unchanging. For many, the idea of mathematical research is hard to comprehend, but this should not be so. Mathematics is dynamic, alive, and growing by leaps and bounds!

2. Look for concrete (especially physical) examples, models, or visualizations of mathematical concepts. Conversely, look for the mathematics in everyday objects. For example, what can you say about the lengths of the chimes hanging beneath a xylophone?

3. Be conscious of the difference between skills and concepts. Skills are sometimes more easily learned, practiced, and tested than concepts—hence very tempting to both students and teachers to spend most of their time on—but a math course that degenerates into a sequence of rote skills and manipulation without any real understanding is essentially worthless.

4. Practice asking lots of questions. A good question is worth a lot more than a mediocre fact.

5. Think about analogies with learning in other disciplines—for example, learning to read, learning how to play a musical instrument. In both cases there is an absolute necessity for constant practice, without which it is impossible to attain any level of sophistication or appreciation. This can be hard work! You cannot read Shakespeare if you are still sounding out individual words letter by letter. Also, under no circumstances would I want my first grade child to be taught to read by a teacher who could only read on an elementary school level, or my twelfth grade child to take an English class with a teacher who could only function on a twelfth grade level. The great teachers have a sense of perspective on their subjects which transcends the level on which they are teaching, and students are acutely aware of this.

6. There is a high probability that you will frequently encounter students that are more adept mathematically than you. You must be prepared for this, or you may tragically close doors of opportunity forever for your students.

7. Your love of mathematics should and will undoubtedly extend to activities outside of the classroom. Great musicians love music and play music, even when they are not performing. Great writers do not confine their creativity to an 8-hour day.

8. You must learn how to learn new mathematics on your own, and teach your students this crucial skill.

9. Mathematics is not a linear subject, but parts are intricately intertwined into a complex structure. In school and college, certain strands are extracted and taught, sometimes giving a very misleading view of mathematics as a whole. Think about ways to reweave the mathematical fabric as you teach.

10. Some of the topics that are studied in a semester took decades, centuries, or even millennia to come to fruition. Is it little wonder that it is hard for students to fully comprehend some of the things we are teaching them the first time (or even the second or third time) that they see them? A good example of this are the concepts of continuity and differentiation in calculus. I recommend that you read about the struggle to try to place the less formal or intuitive views of these concepts on a sound footing. How long did it take?

11. Mathematics is an experimental subject, but the truths discovered are subject to verification in a rigorous way. All too often, the final results are presented in such a way that is cleansed of the explanation of the process of investigation and discovery. This is very sad, for much of value has been lost. One good example of this principle is Archimedes' "Method."

12. High quality work is often hard work. Don't cheat yourself (or others) out of the benefits of significant accomplishment.

There are thousands of books and articles that capture the spirit of mathematics and provide an unimaginable amount of wonderful material. I mention a few of my favorites. There are probably newer editions of all of these.

1. All of the books by Martin Gardner on recreational mathematics and related subjects. Most of this material was first published in the Mathematical Recreations column of *Scientific American*. I grew up on this material starting in elementary school.

2. H. Steinhaus, *Mathematical Snapshots*, Oxford University Press. A gallery of fascinating mathematical snippets.

3. G. Polya, *How to Solve it*, Princeton University Press. My high school teacher showed us a movie by Polya on problem solving that made a deep and long-lasting impression on me.

4. W.W. Rouse Ball and H.S.M. Coxeter, *Mathematical Recreations and Essays*, University of Toronto Press, 1974. I read and reread this book as a high school student.

2 NCTM Curriculum Standards for Grades 9–12

From the website of the National Council of Teachers of Mathematics (http://www.nctm.org).

Standard 1: Mathematics as Problem Solving. In grades 9–12, the mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can—

- use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content;
- apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics;
- recognize and formulate problems from situations within and outside mathematics;
- apply the process of mathematical modeling to real-world problem situations.

Standard 2: Mathematics as Communication. In grades 9–12, the mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can—

- reflect upon and clarify their thinking about mathematical ideas and relationships;
- formulate mathematical definitions and express generalizations discovered through investigations;
- express mathematical ideas orally and in writing;
- read written presentations of mathematics with understanding;
- ask clarifying and extending questions related to mathematics they have read or heard about;
- appreciate the economy, power, and elegance of mathematical notation and its role in the development of mathematical ideas.

Standard 3: Mathematics as Reasoning. In grades 9–12, the mathematics curriculum should include numerous and varied experiences that reinforce and extend logical reasoning skills so that all students can—

- make and test conjectures;

- formulate counterexamples;
- follow logical arguments;
- judge the validity of arguments;
- construct simple valid arguments;

and so that, in addition, college-intending students can—

- construct proofs for mathematical assertions, including indirect proofs and proofs by mathematical induction.

Standard 4: Mathematical Connections. In grades 9–12, the mathematics curriculum should include investigation of the connections and interplay among various mathematical topics and their applications so that all students can—

- recognize equivalent representations of the same concept;
- relate procedures in one representation to procedures in an equivalent representation;
- use and value the connections among mathematical topics;
- use and value the connections between mathematics and other disciplines.

Standard 5: Algebra. In grades 9–12, the mathematics curriculum should include the continued study of algebraic concepts and methods so that all students can—

- represent situations that involve variable quantities with expressions, equations, inequalities, and matrices;
- use tables and graphs as tools to interpret expressions, equations, and inequalities;
- operate on expressions and matrices, and solve equations and inequalities;
- appreciate the power of mathematical abstraction and symbolism;

and so that, in addition, college-intending students can–

- use matrices to solve linear systems;
- demonstrate technical facility with algebraic transformations, including techniques based on the theory of equations.

Standard 6: Functions. In grades 9–12, the mathematics curriculum should include the continued study of functions so that all students can—

- model real-world phenomena with a variety of functions;
- represent and analyze relationships using tables, verbal rules, equations, and graphs;
- translate among tabular, symbolic, and graphical representations of functions;
- recognize that a variety of problem situations can be modeled by the same type of function;
- analyze the effects of parameter changes on the graphs of functions;

and so that, in addition, college-intending students can—

- understand operations on, and the general properties and behavior of, classes of functions.

Standard 7: Geometry From a Synthetic Perspective. In grades 9–12, the mathematics curriculum should include the continued study of the geometry of two and three dimensions so that all students can—

- interpret and draw three-dimensional objects;
- represent problem situations with geometric models and apply properties of figures;
- classify figures in terms of congruence and similarity and apply these relationships;
- deduce properties of, and relationships between, figures from given assumptions;

and so that, in addition, college-intending students can–

- develop an understanding of an axiomatic system through investigating and comparing various geometries.

Standard 8: Geometry From an Algebraic Perspective. In grades 9–12, the mathematics curriculum should include the study of the geometry of two and three dimensions from an algebraic point of view so that all students can—

- translate between synthetic and coordinate representations;
- deduce properties of figures using transformations and using coordinates;
- identify congruent and similar figures using transformations;
- analyze properties of Euclidean transformations and relate translations to vectors;

and so that, in addition, college-intending students can-

- deduce properties of figures using vectors;
- apply transformations, coordinates, and vectors in problem solving.

Standard 9: Trigonometry. In grades 9–12, the mathematics curriculum should include the study of trigonometry so that all students can—

- apply trigonometry to problem situations involving triangles;
- explore periodic real-world phenomena using the sine and cosine functions;

and so that, in addition, college-intending students can—

- understand the connection between trigonometric and circular functions;
- use circular functions to model periodic real-world phenomena;
- apply general graphing techniques to trigonometric functions;
- solve trigonometric equations and verify trigonometric identities;
- understand the connections between trigonometric functions and polar coordinates, complex numbers, and series.

Standard 10: Statistics. In grades 9–12, the mathematics curriculum should include the continued study of data analysis and statistics so that all students can—

- construct and draw inferences from charts, tables, and graphs that summarize data from real-world situations;
- use curve fitting to predict from data;
- understand and apply measures of central tendency, variability, and correlation;
- understand sampling and recognize its role in statistical claims;
- design a statistical experiment to study a problem, conduct the experiment, and interpret and communicate the outcomes;
- analyze the effects of data transformations on measures of central tendency and variability;

and so that, in addition, college-intending students can–

- transform data to aid in data interpretation and prediction;

- test hypotheses using appropriate statistics.

Standard 11: Probability. In grades 9–12, the mathematics curriculum should include the continued study of probability so that all students can—

- use experimental or theoretical probability, as appropriate, to represent and solve problems involving uncertainty;
- use simulations to estimate probabilities;
- understand the concept of a random variable;
- create and interpret discrete probability distributions;
- describe, in general terms, the normal curve and use its properties to answer questions about sets of data that are assumed to be normally distributed;

and so that, in addition, college-intending students can—

- apply the concept of a random variable to generate and interpret probability distributions including binomial, uniform, normal, and chi square.

Standard 12: Discrete Mathematics. In grades 9–12, the mathematics curriculum should include topics from discrete mathematics so that all students can—

- represent problem situations using discrete structures such as finite graphs, matrices, sequences, and recurrence relations;
- represent and analyze finite graphs using matrices;
- develop and analyze algorithms;
- solve enumeration and finite probability problems;

and so that, in addition, college-intending students can—

- represent and solve problems using linear programming and difference equations;
- investigate problem situations that arise in connection with computer validation and the application of algorithms.

Standard 13: Conceptual Underpinnings of Calculus. In grades 9–12, the mathematics curriculum should include the informal exploration of calculus concepts from both a graphical and a numerical perspective so that all students can—

- determine maximum and minimum points of a graph and interpret the results in problem situations;
- investigate limiting processes by examining infinite sequences and series and areas under curves;

and so that, in addition, college-intending students can—

- understand the conceptual foundations of limit, the area under a curve, the rate of change, and the slope of a tangent line, and their applications in other disciplines;
- analyze the graphs of polynomial, rational, radical, and transcendental functions.

Standard 14: Mathematical Structure. In grades 9–12, the mathematics curriculum should include the study of mathematical structure so that all students can—

- compare and contrast the real number system and its various subsystems with regard to their structural characteristics;
- understand the logic of algebraic procedures;
- appreciate that seemingly different mathematical systems may be essentially the same;

and so that, in addition, college-intending students can—

- develop the complex number system and demonstrate facility with its operations;
- prove elementary theorems within various mathematical structures, such as groups and fields;
- develop an understanding of the nature and purpose of axiomatic systems.