MA310 Homework #1
Due Monday, January 25

1. Let's explore what happens when the leading coefficient in a quadratic equation approaches zero. Consider the function $f(x) = ax^2 + 5x - 3$.

   (a) Graphical exploration: On the same coordinate system draw the graph of this function for several values of a between -4 and 4. What is happening to the graph as a approaches zero? What is happening to the x-intercepts?

   (b) Algebraic analysis: As a approaches zero, the formula for $f(x)$ approaches $g(x) = 5x - 3$. What is the x-intercept for $g(x)$? Prove algebraically that one of the x-intercepts for $f(x)$ (i.e., one of the solutions for the quadratic equation $f(x) = 0$) approaches the x-intercept for $g(x)$ as a approaches zero.

   (c) General algebraic analysis: Given a general quadratic equation $ax^2 + bx + c = 0$ where $a \neq 0$, prove that one of the solutions approaches the solution to $bx + c = 0$ as a approaches zero. For convenience, assume that $b > 0$.

2. In this problem we try to find a formula for the square root of a general complex number.

   (a) Find all square roots of i.

   (b) Every complex number $a+bi$ can be represented as a point $P = (a, b)$ in the plane. Assume $a + bi \neq 0$. Let $r = \sqrt{a^2 + b^2}$ and let θ be the angle from the positive x-axis to the ray OP, where O is the origin. Explain why $a+bi = r(\cos\theta + i\sin\theta)$.

   (c) Suppose you want to multiply $a+bi = r(\cos\theta+i\sin\theta)$ by $c+di = r'(\cos\theta'+i\sin\theta')$. Prove that the product is $rr'(\cos(\theta + \theta') + i\sin(\theta + \theta'))$. (I.e., to multiply two complex numbers, multiply their "lengths" and add their "angles..") (Hint: Do you remember your trigonometric angle addition formulas?)

   (d) Prove that one square root of $a+bi = r(\cos\theta+i\sin\theta)$ is $\sqrt{r}(\cos(\theta/2)+i\sin(\theta/2))$. What is the other square root? How can you find you find them graphically?

   (e) Find all square roots of $1 + i$. Express your final answer without using sines and cosines. (Hint: Do you remember your trigonometric half-angle formulas?)

   (f) Find a general formula for the square roots of $a + bi$ in terms of a and b. For convenience, assume that $a + bi \neq 0$ and $b \geq 0$. Suggestion: Use the half-angle formulas and the fact that $a = r\cos\theta$ and $b = r\sin\theta$. Confirm your answer by squaring it.