

# MA 310 — Homework #1

## Solution

Play the “Game of 100” (see “Problems” on the course website) with others and try to formulate a playing strategy. Write down what you come up with, even if it is incomplete.

**Solution.** We claim that the set of numbers of the form  $2 + 7\ell$ ,  $\ell = 0, 1, 2, \dots, 14$ , are “good” positions (ones you wish to reach), while the other numbers from 0 to 100 are “bad” (ones you do not wish to reach), in the following sense: From a good position all single moves lead to bad positions, and from a bad position there exists a single move to a good position. (Note that 100 is a good position.)

For suppose you achieve a position of the form  $2 + 7j$ . Then on his/her next move, your opponent can only achieve positions  $3 + 7j$ ,  $4 + 7j$ ,  $5 + 7j$ ,  $6 + 7j$ ,  $7 + 7j = 7(j + 1)$ , or  $1 + 7(j + 1)$ , and none of these are of the form  $2 + 7\ell$ . On the other hand, from each of these bad positions you can make a single move (adding 6, 5, 4, 3, 2, 1, respectively), and reach the position  $2 + 7(\ell + 1)$ , another good position.

Thus the first player can guarantee a win by moving to 2 and subsequently reaching good positions.