

MA 310 — Homework #10

Solutions

Exam #3 will be on Wednesday, April 22.

1. Solve “Repeating Decimals”.

Solution. At some point when carrying out your long division you will be “bringing down” zeros each time. Your remainder at each step is an integer from 0 to $b - 1$. If the remainder is ever 0, then you have a terminating decimal. If the remainder is never zero, then at some point you will encounter a remainder that you saw before, and carry out the same step as before, and hence continue repeating certain previous steps. Since the remainders you can encounter are only 1 through $b - 1$, the repetition never exceeds $b - 1$ divisions.

2. Solve “Five Points in a Triangle”.

Solution. Divide the triangle in the natural way into four equilateral triangles each of side length $1/2$. By the pigeonhole principle, there must be at least two points contained in the same smaller triangle, and so these two points can be no farther than $1/2$ unit apart.

3. Solve “Faces in a Polyhedron”.

Let F be a polygon with the maximum number of edges. Call this maximum number n . Then F has n neighboring faces, so the polyhedron has at least $n + 1$ faces, each of which has at most n edges. By the pigeonhole principle, at least two of these $n + 1$ faces must have the same number of edges.

4. Solve “Lattice Points on Line Segments”.

Solution. Each lattice point (x, y) falls into one of four types, depending upon whether x is even or odd and y is even or odd. By the pigeonhole principle, there must be at least two lattice points (x_1, y_1) and (x_2, y_2) of the same type. But then the midpoint $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$ of the line segment joining these two points will have integer coordinates, since the average of two even numbers is an integer, and the average of two odd numbers is an integer.