MA 341 Homework #8
Due Monday, November 24, in Class

1. Find all complex numbers z such that $z^3 = -8i$.

2. Course Notes, Problem 7.2.7. Take pains to make neat, clear diagrams.

3. Prove that ℓ_1 and ℓ_2 are parallel lines, then the net effect of first reflecting across ℓ_1 and then reflecting across ℓ_2 is a translation in the direction perpendicular to the lines, directed from ℓ_1 towards ℓ_2, by an amount equal to twice the distance between the two lines.

4. (a) Consider the circles C_1 described by $(x-a_1)^2+(y-b_1)^2=c_1^2$ and C_2 described by $(x-a_2)^2+(y-b_2)^2=c_2^2$. Prove algebraically that C_1 and C_2 can share at most two points, and further, if they do share two different points P and Q, then the perpendicular bisector of the segment $\overline{PQ}$ is the line through the centers of the circles.

   (b) Let f be an isometry of the plane (not necessarily one of the four specific types we have been discussing). Let A, B, C be three noncollinear points. Show that if you know $f(A)$, $f(B)$, and $f(C)$, then you can determine $f(P)$ for any point. That is to say, f is uniquely determined by its action on any three particular noncollinear points.

5. Consider the set of points S in the plane (a "strip") described by $S = \{(x,y) \in \mathbf{R}^2 : -1 \le y \le 1\}$. Carefully describe the set of all translations, rotations, reflections, and glide reflections that map every point in S back into S.