Transformation Matrices

1. Translation by the vector (p, q).

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & p \\ 0 & 1 & q \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

   Note: If $(p, q) = (0, 0)$, then this is just the identity matrix.

2. Counterclockwise rotation by the angle δ about the point (p, q).

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} c & -s & -pc + qs + p \\ s & c & -ps - qc + q \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

   where $c = \cos\delta$ and $s = \sin\delta$.

   Note: If $\delta = 0$, then this is just the identity matrix.

3. Reflection across the line $px + qy = r$, where without loss of generality we can assume $p^2 + q^2 = 1$. IMPORTANT: Before applying this formula, rescale the equation of the line, if necessary. If the equation of the line is $ax + by = c$, first divide through by $\sqrt{a^2 + b^2}$ to get the equation in the form $px + qy = r$ in which $p^2 + q^2 = 1$.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 - 2p^2 & -2pq & 2pr \\ -2pq & 1 - 2q^2 & 2qr \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

4. Glide reflection across the line $px + qy = r$ using the translation $(tq, -tp)$, where without loss of generality we can assume $p^2 + q^2 = 1$. IMPORTANT: Before applying this formula, rescale the equation of the line, if necessary. If the equation of the line is $ax + by = c$, first divide through by $\sqrt{a^2 + b^2}$ to get the equation in the form $px + qy = r$ in which $p^2 + q^2 = 1$.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 - 2p^2 & -2pq & 2pr + tq \\ -2pq & 1 - 2q^2 & 2qr - tp \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

   Note: If $t = 0$ then this is just a pure reflection.