MA515 Homework #8
Due Wednesday, November 10

Problems to hand in:

1. Problem (Finding a negative-weight dicycle), p. 77.

2. Read the definition of a matrix being totally unimodular in the beginning of Section 0.8. Do the following:

   (a) Prove that if any one of the following matrices are totally unimodular, then they all are:

      i. A.
      ii. $-A$.
      iii. A^T.
      iv. $[A, A]$ (two copies of A side by side).
      v. $[A, I]$ (a copy of A and a copy of I side by side).

   (b) Assume that a matrix A is such that

      i. Every entry of A is either 0, 1, or -1.
      ii. Every column of A contains at most two nonzero entries.
      iii. The rows of A can be partitioned into two subsets R_1 and R_2 with the property that for every column of A with two nonzero entries, if the two entries have the same sign then one is in R_1 and the other is in R_2, and if the two entries have opposite signs, then both are in R_1 or both are in R_2.

      Prove that A is totally unimodular. Suggestion: Modify the proof of the first theorem on page 44.

   (c) Problem (Unimodularity and pivoting), p. 43.

   (d) An undirected graph is bipartite if its vertices can be partitioned into two subsets V_1 and V_2 such that every edge of G has one endpoint in V_1 and the other in V_2. The vertex-edge incidence matrix of an undirected graph G is the matrix B, with rows indexed by the vertices of G, the columns indexed by the edges of G, and the entry in row v column e being 1 if v is an endpoint of e and 0 otherwise.

      i. Prove that an undirected graph is bipartite if and only if it contains no cycles with an odd number of edges.
      ii. Prove or disprove: An undirected graph is bipartite if and only if its vertex-edge incidence matrix is totally unimodular.

Problems to work but not hand in:

1. Carefully go through the proof of Dijkstra's algorithm in the book, pp. 79–81.
2. Exercise (Bellman-Ford Algorithm), p. 76. Now apply Floyd-Warshall to this graph.