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phase1.ms
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Example of Phase I--Phase II from Chvatal.  Here is the
original LP:

max x1-x2+x3
st 2x1-x2+2x3<=4
2x1-3x2+x3<=-5
-x1+x2-2x3<=-1
x1,x2,x3>=0

The Phase I problem is:

max w=-x0
st 2x1-x2+2x3+x4-x0=4
2x1-3x2+x3+x5-x0=-5
-x1+x2-2x3+x6-x0=-1
x0,x1,x2,x3,x4,x5,x6>=0

Here is the initial table, with basis (4,5,6):
> with(linalg):
Warning: new definition for   norm
Warning: new definition for   trace

> T1:=matrix([
  [x1,x2,x3,x4,x5,x6,x0,-w,RHS],
  [2,-1,2,1,0,0,-1,0,4],
  [2,-3,1,0,1,0,-1,0,-5],
  [-1,1,-2,0,0,1,-1,0,-1],
  [0,0,0,0,0,0,-1,1,0]
  ]);
```

$$T1 := \begin{bmatrix} x1 & x2 & x3 & x4 & x5 & x6 & x0 & -w & RHS \\ 2 & -1 & 2 & 1 & 0 & 0 & -1 & 0 & 4 \\ 2 & -3 & 1 & 0 & 1 & 0 & -1 & 0 & -5 \\ -1 & 1 & -2 & 0 & 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix}$$

```
Pivot x0 in, x5 out, to get a primal feasible table with
basis (4,0,6):
> T2:=pivot(mulrow(T1,3,-1),3,7,2..5);
```

$$T2 := \begin{bmatrix} x1 & x2 & x3 & x4 & x5 & x6 & x0 & -w & RHS \\ 0 & 2 & 1 & 1 & -1 & 0 & 0 & 0 & 9 \\ -2 & 3 & -1 & 0 & -1 & 0 & 1 & 0 & 5 \\ -3 & 4 & -3 & 0 & -1 & 1 & 0 & 0 & 4 \\ -2 & 3 & -1 & 0 & -1 & 0 & 0 & 1 & 5 \end{bmatrix}$$

```
  Pivot x2 in, x6 out, to get basis (4,0,2):
> T3:=pivot(mulrow(T2,4,1/4),4,2,2..5);
```

$$T3 := \begin{bmatrix} x1 & x2 & x3 & x4 & x5 & x6 & x0 & -w & RHS \\ \frac{3}{2} & 0 & \frac{5}{2} & 1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 & 7 \\ \frac{1}{4} & 0 & \frac{5}{4} & 0 & \frac{-1}{4} & \frac{-3}{4} & 1 & 0 & 2 \\ \frac{-3}{4} & 1 & \frac{-3}{4} & 0 & \frac{-1}{4} & \frac{1}{4} & 0 & 0 & 1 \\ \frac{1}{4} & 0 & \frac{5}{4} & 0 & \frac{-1}{4} & \frac{-3}{4} & 0 & 1 & 2 \end{bmatrix}$$

```
  Pivot x3 in, x0 out, to get basis (4,3,2):
> T4:=pivot(mulrow(T3,3,4/5),3,3,2..5);
```

$$T4 := \begin{bmatrix} x1 & x2 & x3 & x4 & x5 & x6 & x0 & -w & RHS \\ 1 & 0 & 0 & 1 & 0 & 1 & -2 & 0 & 3 \\ \frac{1}{5} & 0 & 1 & 0 & \frac{-1}{5} & \frac{-3}{5} & \frac{4}{5} & 0 & \frac{8}{5} \\ \frac{-3}{5} & 1 & 0 & 0 & \frac{-2}{5} & \frac{-1}{5} & \frac{3}{5} & 0 & \frac{11}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix}$$

This is an optimal table for the Phase I problem. The original LP is feasible since x0=0. We now begin Phase II by deleting the artificial variable x0 and replacing the last row with the original objective function:

```
> T4[1,8]:=-z;
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$$T4_{1,8} := -z$$

```
> T4a:=delrows(T4,5..5);
```

$$T4a := \begin{bmatrix} x1 & x2 & x3 & x4 & x5 & x6 & x0 & -z & RHS \\ 1 & 0 & 0 & 1 & 0 & 1 & -2 & 0 & 3 \\ \frac{1}{5} & 0 & 1 & 0 & \frac{-1}{5} & \frac{-3}{5} & \frac{4}{5} & 0 & \frac{8}{5} \\ \frac{-3}{5} & 1 & 0 & 0 & \frac{-2}{5} & \frac{-1}{5} & \frac{3}{5} & 0 & \frac{11}{5} \end{bmatrix}$$

```
> T4b:=delcols(T4a,7..7);
```

$$T4b := \begin{bmatrix} x1 & x2 & x3 & x4 & x5 & x6 & -z & RHS \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 3 \\ \frac{1}{5} & 0 & 1 & 0 & \frac{-1}{5} & \frac{-3}{5} & 0 & \frac{8}{5} \\ \frac{-3}{5} & 1 & 0 & 0 & \frac{-2}{5} & \frac{-1}{5} & 0 & \frac{11}{5} \end{bmatrix}$$

```
> T5:=stack(T4b,matrix([[1,-1,1,0,0,0,1,0]]));
```

$$T5 := \begin{bmatrix} x1 & x2 & x3 & x4 & x5 & x6 & -z & RHS \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 3 \\ \frac{1}{5} & 0 & 1 & 0 & \frac{-1}{5} & \frac{-3}{5} & 0 & \frac{8}{5} \\ \frac{-3}{5} & 1 & 0 & 0 & \frac{-2}{5} & \frac{-1}{5} & 0 & \frac{11}{5} \\ 1 & -1 & 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Repivot in basic columns to restore identity matrix:

```
> T5a:=pivot(T5,2,4,2..5):
  T5b:=pivot(T5a,3,3,2..5):
  T6:=pivot(T5b,4,2,2..5);
```

$$T6 := \begin{bmatrix} x1 & x2 & x3 & x4 & x5 & x6 & -z & RHS \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 3 \\ \frac{1}{5} & 0 & 1 & 0 & \frac{-1}{5} & \frac{-3}{5} & 0 & \frac{8}{5} \\ \frac{-3}{5} & 1 & 0 & 0 & \frac{-2}{5} & \frac{-1}{5} & 0 & \frac{11}{5} \\ \frac{1}{5} & 0 & 0 & 0 & \frac{-1}{5} & \frac{2}{5} & 1 & \frac{3}{5} \end{bmatrix}$$

```
  The basis is still (4,3,2).  Begin Phase II.  Pivot x6 in, x4
  out, to get basis (6,3,2):
> T7:=pivot(T6,2,6,2..5);
```

$$T7 := \begin{bmatrix} x1 & x2 & x3 & x4 & x5 & x6 & -z & RHS \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 3 \\ \frac{4}{5} & 0 & 1 & \frac{3}{5} & \frac{-1}{5} & 0 & 0 & \frac{17}{5} \\ \frac{-2}{5} & 1 & 0 & \frac{1}{5} & \frac{-2}{5} & 0 & 0 & \frac{14}{5} \\ \frac{-1}{5} & 0 & 0 & \frac{-2}{5} & \frac{-1}{5} & 0 & 1 & \frac{-3}{5} \end{bmatrix}$$

```
>
  This is an optimal table.  The optimal solution is
  (0,14/5,17/5,0,0,3) with optimal value 3/5.
```