

MA 109: September 20

Average Rate of Change: With Variables

Start of Class

Instructor Information

Name:

Email:

Office Hours:

Warm-up Questions

Notes

The process for finding Average Rate of Change is always the same:

- Both of the given values are x-values They could be:
 - all numbers
 - numbers and variables
 - all variables
- Plug the given x-values in to get the y-values
- Plug all of that into the slope formula to get AROC

Example: Suppose $f(x) = x^2 + 5$. What is the average rate of change of $f(x)$ on $[-2, -2+h]$?

\uparrow \uparrow
 x_1 x_2

$$y_1 = f(x_1) = f(-2) = (-2)^2 + 5 = 4 + 5 = 9$$

$(-2, 9)$

$$y_2 = f(x_2) = f(-2+h) = (-2+h)^2 + 5$$

remember: "squaring" means "multiplying by itself"

$$= (-2+h)(-2+h) + 5$$
$$= 4 - 2h - 2h + h^2 + 5$$
$$= -4h + h^2 + 9$$

$(-2+h, -4h + h^2 + 9)$

$$\text{AROC} = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-4h + h^2 + 9) - (9)}{(-2+h) - (-2)}$$
$$= \frac{-4h + h^2 + \cancel{9} - \cancel{9}}{\cancel{-2} + h + \cancel{2}}$$
$$= \frac{-4h + h^2}{h}$$
$$= \frac{\cancel{h}(-4 + h)}{\cancel{h}}$$
$$= \boxed{-4 + h}$$

Example: Suppose $f(x) = x^2 + 5$. What is the average rate of change of $f(x)$ on $[a, a + h]$?

x_1 \uparrow \uparrow
 x_2

$$y_1 = f(x_1) = f(a) = a^2 + 5$$

$(a, a^2 + 5)$

$$y_2 = f(x_2) = f(a+h) = (a+h)^2 + 5$$
$$= (a+h)(a+h) + 5$$
$$= a^2 + ah + ah + h^2 + 5$$
$$= a^2 + 2ah + h^2 + 5$$

$(a+h, a^2 + 2ah + h^2 + 5)$

$$\text{AROC} = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(a^2 + 2ah + h^2 + 5) - (a^2 + 5)}{(a+h) - (a)}$$
$$= \frac{\cancel{a^2} + 2ah + \cancel{h^2} + \cancel{5} - \cancel{a^2} - \cancel{5}}{\cancel{a+h} - \cancel{a}}$$
$$= \frac{2ah + h^2}{h}$$
$$= \frac{\cancel{h}(2a+h)}{\cancel{h}}$$
$$= \boxed{2a+h}$$

End of Class

Write a summary of what you learned today:

What questions do you have about the material from today?

What do you need to do between now and the next class meeting?