

# MA 109: September 19

Average Rate of Change

Start of Class

Instructor Information

Name:

Email:

Office Hours:

Warm-up Questions

## Notes

Average rate of change is slope

Example: Suppose  $f(x) = 3x^2 - 4$ . What is the average rate of change of  $f(x)$  on  $[-1, 2]$ ?  
 $x_1$   $x_2$

Strategy: plug in the  $x$ 's to get the  $y$ 's and then plug into the slope formula

$$y_1 = f(x_1) = f(-1) = 3(-1)^2 - 4 = 3(1) - 4 = 3 - 4 = -1$$

$(-1, -1)$

$$y_2 = f(x_2) = f(2) = 3(2)^2 - 4 = 3(4) - 4 = 12 - 4 = 8$$

$(2, 8)$

$$AROC = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - (-1)}{2 - (-1)} = \frac{8 + 1}{2 + 1} = \frac{9}{3} = \boxed{3}$$

The process for finding Average Rate of Change is always the same:

① The two given values are always the  $x$ -values They could be:

- both numbers
- numbers and variables
- both variables

② plug the  $x$ -values into the function to get the  $y$ -values

③ plug all of that into the slope formula and simplify

Example: Suppose  $f(x) = x^2 + 5$ . What is the average rate of change of  $f(x)$  on  $[-2, -2+h]$ ?  $x_1$   $x_2$

$$y_1 = f(x_1) = f(-2) = (-2)^2 + 5 = 4 + 5 = 9 \quad (-2, 9)$$

$$y_2 = f(x_2) = f(-2+h) = (-2+h)^2 + 5$$

remember: "squaring" means "multiplying by itself"

$$= (2+h)(-2+h) + 5$$

$$= 4 - 2h - 2h + h^2 + 5$$

$$= -4h + h^2 + 9$$

$$(-2+h, -4h + h^2 + 9)$$

$$\text{AROC} = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-4h + h^2 + 9) - (9)}{(-2+h) - (-2)}$$

$$= \frac{-4h + h^2 + \cancel{9} - \cancel{9}}{\cancel{-2} + h + \cancel{2}}$$

$$= \frac{-4h + h^2}{h}$$

$$= \frac{\cancel{h}(-4+h)}{\cancel{h}}$$

$$= \boxed{-4+h}$$

Example: Suppose  $f(x) = x^2 + 5$ . What is the average rate of change of  $f(x)$  on  $[a, a + h]$ ? x<sub>1</sub> x<sub>2</sub>

$$y_1 = f(x_1) = f(a) = a^2 + 5 \quad (a, a^2 + 5)$$

$$y_2 = f(x_2) = f(a+h) = (a+h)^2 + 5$$

$$= (a+h)(a+h) + 5$$

$$= a^2 + ah + ah + h^2 + 5$$

$$= a^2 + 2ah + h^2 + 5$$

$$(a+h, a^2 + 2ah + h^2 + 5)$$

$$\text{AROC} = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(a^2 + 2ah + h^2 + 5) - (a^2 + 5)}{(a+h) - (a)}$$

$$= \frac{\cancel{a^2} + 2ah + \cancel{h^2} + \cancel{5} - \cancel{a^2} - \cancel{5}}{\cancel{a} + h - \cancel{a}}$$

$$= \frac{2ah + h^2}{h}$$

$$= \frac{\cancel{h}(2a+h)}{\cancel{h}}$$

$$= \boxed{2ah}$$

## End of Class

Write a summary of what you learned today:

What questions do you have about the material from today?

What do you need to do between now and the next class meeting?