Today's Goal:

So far we have been studying polynomial functions graphically. to study polynomials algebraically. Most of our work will be concerned with factoring polynomials, and to factor, we need to know how to divide polynomials.

Assignments:

Homework (Sec. 4.2): # 1,3,5,11,13,19,22,27,31,36,43,53 (pp. 331-332).

Given the integers 23 and 5 we can 'divide' one by the other. We obtain: $\frac{23}{5} = 4 + \frac{3}{5}$ or $23 = 4 \cdot 5 + 3$. In general, if a and b are non-zero integers, then there exist unique integers q and r such that

$$a = q \cdot b + r$$
 and $0 < r < |b|$,

where q is the quotient and r the remainder. This is the usual 'long division' familiar from elementary arithmetic.

Example 1: Divide 63 by 12.

$$\frac{63}{12} = 5 + \frac{3}{12} \qquad \text{OR} \qquad 63 = 5 \cdot 12 + 3$$

▶ Long Division of Polynomials: Dividing polynomials is much like the familiar process of dividing numbers. This process is the long division algorithm for polynomials.

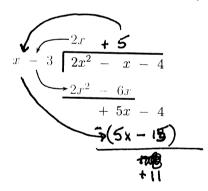
Division Algorithm: If P(x) and D(x) are polynomials, with $D(x) \neq 0$, then there exist unique polynomials mials Q(x) and R(x), where R(x) is either 0 or of degree strictly less than the degree of D(x), such that

$$P(x) = Q(x) \cdot D(x) + R(x)$$

The polynomials P(x) and D(x) are called the **dividend** and **divisor**, respectively; Q(x) is the **quotient** and R(x) is the **remainder**.

Example 2: Divide the polynomial

$$P(x) = 2x^2 - x - 4$$
 by $D(x) = x - 3$.



$$2x^{2}-x-4=(2x+5)(x-3)+11$$

$$\angle$$
: $(2x+5)(x-3) + $1=$

$$2x^2 - 6x + 5x - 15 + $1=$$

$$2x^2 - x - 4$$

(Complete the above table and check your work!)

Example 3: Divide the polynomial

$$P(x) = x^4 - x^3 + 4x + 2$$
 by $D(x) = x^2 + 3$.

$$\begin{array}{r}
x^{2} - x - 3 \\
x^{4} - x^{3} + 6x^{2} + 4x + 2 \\
-(x^{4} + 0x^{3} + 3x^{2}) \\
\hline
-x^{3} + 3x^{2} + 4x \\
-(-x^{3} - 0x^{2} - 3x) \\
\hline
-3x^{2} + 7x + 2 \\
(-3x^{2} - 0x - 9) \\
\hline
7x + 11
\end{array}$$

$$x^{4}-x^{3}+4x+2=(x^{2}-x-3)(x^{2}+3)+(7x+11)$$

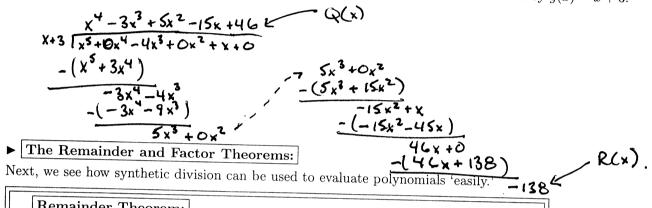
$$\underline{\checkmark:} (x^{2}-x-3)(x^{2}+3)+7x+11=$$

$$x^{4}-x^{3}-3x^{2}+3x^{2}-3x-9+7x+11=$$

$$x^{4}-x^{3}+4x+2$$

Example 4:

Find the quotient Q(x) and the remainder R(x) when $f(x) = x^5 - 4x^3 + x$ is divided by g(x) = x + 3.



Remainder Theorem:

If the polynomial P(x) is divided by x-c, then the remainder is the value P(c).

Proof: If the divisor D(x) is of the form x-c, then the remainder MUST be a constant R. Thus

$$P(x) = Q(x) \cdot (x - c) + R.$$

Setting x = c in the above equation gives that $P(c) = Q(c) \cdot 0 + R = R$. Thus

$$P(x) = Q(x) \cdot (x - c) + P(c).$$

From the boxed equation we obtain our next theorem, which says that the zeros of a polynomial correspond to the linear factors of the polynomial.

Factor Theorem:

The number c is a zero of P(x) if and only if x - c is a factor of P(x); that is, $P(x) = Q(x) \cdot (x - c)$ for some polynomial Q(x).

Example 6: Let $P(x) = x^3 + 2x^2 - 7$

- (a) Find the quotient and the remainder when P(x) is divided by x + 2.
- (b) Use the Remainder Theorem to find P(-2).

Theorem to find
$$P(-2)$$
.

(a) $\frac{\chi^2}{\chi^3 + 2\chi^2 + 0\chi - 7}$
 $\frac{-(\chi^3 + 2\chi^2)}{-7}$
So $\chi^3 + 2\chi^2 - 7 = \chi^2(\chi + 2) - 7$

(b) By THE REMAINDER THAT.,
$$P(-2) = -7$$

 $\sqrt{:}$ $P(-2) = (-2)^3 + 2(-2)^2 - 7 = -8 + 8 - 7 = -7$.

Use the Factor Theorem to determine whether x + 2 is a factor of $f(x) = 3x^6 + 2x^3 - 176$. Example 7:

According to the factor theorem, x+2 is a factor of f(x) IF AND ONLY IF -2 IS 1 ZERO OF F(x), i.e. F(-2)=0.

$$f(-z) = 3(-z)^4 + 2(-z)^3 - 176$$

= $3(+L4) + 2(-8) - 176$
= $192 - 16 - 176$
= $192 - 192 = 0$
 $f(x) = 3x^6 + 2x^3 - 176$

Example 8:

Find a polynomial of degree 3 that has zeros 1, -2, and 3, and in which the coefficient of x^2 is 3.

IF your polynomiae HAS Zeros 1, -2 AND 3, IT HAS FACTORS

 $= x^3 - 7x^2 - 5x + 6$ Now if we want the coefficient of x2 to be 3, we need to multiple $\left(-\frac{3}{2}\right)(x-1)(x+2)(x-3) = -\frac{3}{2}(x^3-2x^2-5x+6)$

Let $P(x) = 2x^3 + 3x^2 - 17x - 30$.

$$= -\frac{3}{2}x^3 + 3x^2 + \frac{15}{2}x - 9.$$

- Is 3 a zero of P(x)? What does this tell you about the factors of P(x)? What does it tell you about the graph of y = P(x)?
- Is 2 a zero of P(x)? What does this tell you about the factors of P(x)? What does it tell you about the graph of y = P(x)?
- · 1:3 4 zero of P(x)? P(3)= Z(3)3+3(3)2-17(3)-30= 54+27-51-30=0 Yes, 3 is a zero of PGD. This means (x-3) is a factor of PCX) AND THE graph of y= P(x) touches or crosses the x-Axis AT x=3.
- 15 2 A zero of P(x)? P(z)=2(z)3+3(z)2-17(z)-30=16+12-34-30=28-64=-36 +0. No. 2 is NOT A ZERO of PCX).

So (x-2) IS NOT 4 FACTOR OF P(x) AND THE graph Example 10: of y = P(x) does not touch or closs the x-Ax of x = Z. The graph of a polynomial has x-intercepts at (2,0) and (-5,0). What does this tell you about the polynomial?

This means x=2 AND x=-5 ARE Zeros of - THIS polynomials Which, in tuen, means (x-2) and (x-(5)) are factors Of the polynomia.

Remember x-(-5) = x+5