detailed

MA109 — College Algebra	Fall 2015	Name: KEY	Sec.:
Exam 3	2015-11-18		

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

Problem	Answer	Problem	Answer
1.	a	11.	d
2.	d	12.	d
3.	c	13.	a
4.	b	14.	c
5.	b	15.	e
6.	b	16.	c
7.	e	17.	d
8.	e	18.	a
9.	b	19.	e
10.	c	20.	a

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Name: ________________

Multiple Choice Questions

Show all your work on the page where the question appears. Clearly mark your answer both on the cover page on this exam and in the corresponding questions that follow.

1. Find the indicated value of the function when $x = \sqrt{6} + 2$.

* replace every x in function rule

$$f(x) = \sqrt{\boxed{x} + 8} - \boxed{x} - 3$$

$f(\sqrt{6} + 2) =$

$f(\sqrt{6}+2) = \sqrt{(\sqrt{6}+2) + 8} - (\sqrt{6}+2) - 3$

$f(\sqrt{6}+2) = \sqrt{\sqrt{6}+2+8} - \sqrt{6} - 2 - 3$

$\boxed{f(\sqrt{6}+2) = \sqrt{\sqrt{6}+10} - \sqrt{6} - 5}$

Possibilities:

(a) $\sqrt{\sqrt{6} + 10} - \sqrt{6} - 5$

(b) 5

(c) $\sqrt{10} - 5$

(d) $\sqrt{\sqrt{6} + 10} - \sqrt{6} - 1$

(e) $\sqrt{16} - \sqrt{6} - 5$

* simplify by combining like terms

2. Find $f(4)$ if $f(x) = \begin{cases} 8 & \text{if } x \leq 1 \\ 2x + 6 & \text{if } 1 < x \leq 3 \\ 3\boxed{x} + 3 & \text{if } 3 < x \leq 5 \\ 18 & \text{if } x > 5 \end{cases}$

input

$3 < 4 \leq 5$

* first, determine which of the function rules apply for input value

Possibilities:

(a) 18

(b) 8

(c) 12

(d) 15

(e) 14

$f(4) = 3(4) + 3$

$= 12 + 3$

$= \boxed{15}$

* replace every x in the appropriate rule with the input value

* simplify

3. Find the domain of $\sqrt[3]{\frac{x-3}{7}}$

Possibilities:

(a) $(-\infty, 3) \cup (3, \infty)$
(b) $(3, \infty)$
(c) $(-\infty, \infty)$
(d) $[\frac{3}{7}, \infty)$
(e) $[3, \infty)$

* Domain restriction red flags, at this point in semester

to avoid non-real values ⟹ → variables under even roots

to avoid dividing by zero ⟹ → variables in denominator

* root is odd, so no restriction

* denominator has no variables, so no restrictions

* no restrictions, so $\boxed{\text{all real \#s OR } (-\infty,\infty)}$

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4. Find the domain of $\frac{3}{\sqrt{x-7}}$

Possibilities:

(a) $(-\infty, 7) \cup (7, \infty)$
(b) $(7, \infty)$
(c) $[\frac{3}{7}, \infty)$
(d) $[7, \infty)$
(e) $(-\infty, \infty)$

* Is there an even root?
avoiding negative values under root

* Is there a variable in denominator?
avoiding zero in denominator

$x - 7 > 0$

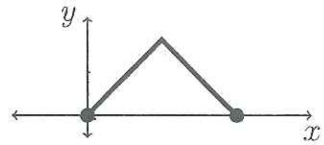
$x > 7$

NOTE: cannot equal zero in this case because expression is in the denominator of fraction!

7

$\boxed{(7, \infty)}$

5. Which situation below is most reasonable depicted in this graph:



* determine growth pattern for each given scenario

Possibilities:

(a) y is the how many days are left in the semester after x weeks of school, if $x = 0$ is the first week of class.

(b) y is the distance from home at time x as you run to the end of the block and back at a steady pace.

(c) y is the temperature of left-over food at time x if the food is placed in the refrigerator at time $x = 0$.

(d) y is the number of bacteria at time x if the bacteria experience a steady rate of exponential growth.

(e) y is the temperature of an oven at time x if it switched on at time $x = 0$ and left on.

~~a)~~ the number of days left is always decreasing

*⇒ b) the distance from home would increase until you started back

~~c)~~ the temperature would decrease until it reached fridge temp

~~d)~~ the number of bacteria would increase exponentially

~~e)~~ the temperature would increase until it reached oven temp

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6. A car moves along a straight test track. The distance traveled by the car at various times is shown in the table. Find the average speed of the car from 10 to 15 seconds.

Time (seconds)	0	5	10	15	20	25	30
Distance (feet)	0	50	200	450	800	1250	1800

* Average speed is $\dfrac{\text{change in distance}}{\text{change in time}}$

Possibilities:

(a) 20 feet per second

(b) 50 feet per second

(c) 80 feet per second

(d) 60 feet per second

(e) 30 feet per second

from table

specified in question

$$\text{Avg. Speed} = \frac{450 - 200}{15 - 10}$$

$$= \frac{250 \leftarrow \text{feet}}{5 \leftarrow \text{seconds}}$$

$$\boxed{= 50 \text{ feet per second}}$$

7. Simplify the formula for the average rate of change of $f(x) = (x-3)^2 + 7$ from $x = 3$ to $x = 3 + h$

"a" "b"

Possibilities: * A.R.O.C. formula $\dfrac{f(b) - f(a)}{b - a}$

(a) $3 + 2h$

(b) $2h$

(c) $6 + h$

(d) 1

(e) h

$$\text{A.R.O.C.} = \frac{\overset{f(b)}{f(3+h)} - \overset{f(a)}{f(3)}}{\underset{b}{(3+h)} - \underset{a}{3}}$$

$$= \frac{[(\boxed{3+h}-3)^2+7] - [(\boxed{3}-3)^2+7]}{3+h-3}$$

plug 3+h in for x

plug 3 in for x

$$= \frac{[(h+0)^2+7] - [0^2+7]}{h}$$

$$= \frac{h^2 + 7 - 7}{h}$$

$$= \frac{h^2}{h}$$

$$\boxed{= h}$$

Simplifying

* plug in values for a & b into function & simplify

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8. Find the domain of $\left(\dfrac{f}{g}\right)(x)$ if $f(x) = 3x^2 + 7x + 8$ and $g(x) = 2x - 9$

Possibilities:

(a) $[\frac{9}{2}, \infty)$

(b) $\left[\frac{-7 \pm \sqrt{7^2 - 4(3)(8)}}{6}, \infty\right)$

(c) $(-\infty, \frac{2}{9})$

(d) $(-\infty, \infty)$

(e) $(-\infty, \frac{9}{2}) \cup (\frac{9}{2}, \infty)$

* Domain for $\left(\frac{f}{g}\right)(x) \Rightarrow x$ in domain of f
$\Rightarrow x$ in domain of g
$\Rightarrow g(x) \neq 0$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x^2+7x+8}{2x-9}$$

no fraction & no log
no even root

Domain of f: polynomial $\Rightarrow$ no restrictions

Domain of g: polynomial $\Rightarrow$ no restrictions

same here

Domain of $\frac{f}{g}$: $g(x) \neq 0$
$2x - 9 \neq 0$
$2x \neq 9$
$x \neq \frac{9}{2}$

only restriction for quotient

$\frac{9}{2}$

$\boxed{(-\infty, \frac{9}{2}) \cup (\frac{9}{2}, \infty)}$

9. Find $(f-g)(6)$ where $f(x) = 4x^2 - 8x - 9$ and $g(x) = 3x - 2$

Possibilities:

(a) 103

(b) 71

(c) 259

(d) 67

(e) 887

* Subtracting functions is like subtracting the outputs $\Longrightarrow (f-g)(x) = f(x) - g(x)$

$(f-g)(6) = f(6) - g(6) \leftarrow$ plug 6 in for x

$= \underbrace{[4(\boxed{6})^2 - 8(\boxed{6}) - 9]}_{f(6)} - \underbrace{[3(\boxed{6}) - 2]}_{g(6)}$

simplify using order of operations

$= [4(36) - 48 - 9] - [18 - 2]$

$= [87] - [16]$

$\boxed{= 71}$

10. Simplify the formula for $(f \circ g)(x)$ if $f(x) = 1 - x$ and $g(x) = \frac{x-1}{x}$

Hint: try plugging in $x = 9$

Possibilities:

(a) $\dfrac{x}{x-1}$

(b) $9x$

(c) $\dfrac{1}{x}$

(d) x

(e) $\dfrac{9}{x}$

* Composition of functions:

$(f \circ g)(x) = f(\ g(x)\ )$ ← plug the g function in for x in the f function

$(f \circ g)(x) = f(\ g(x)\ )$

$= f\left(\ \frac{x-1}{x}\ \right)$

$= 1 - \boxed{\frac{x-1}{x}}$

$= \frac{x}{x} - \frac{x-1}{x}$ ← get common denominator

$= \frac{x-(x-1)}{x}$ ← distribute the negative

$= \frac{x-x+1}{x}$ ← simplify

$\boxed{= \frac{1}{x}}$

USING THE HINT:

$(f \circ g)(9) = f(\ g(9)\ )$

$= f\left(\ \frac{9-1}{9}\ \right)$

$= f\left(\frac{8}{9}\right)$

$= 1 - \frac{8}{9}$

$= \frac{1}{9}$

11. Suppose that the graph of $y = f(x)$ contains the point $(4, 8)$. Find a point that must be on the graph of $y = g(x)$ for $g(x) = \underbrace{9+}\,f(\underbrace{3x+2})$.

*affects INPUT, or x, values

*affects OUTPUT, or y, values

set new input expression = to old input value

evaluate new function w/ new input value

Possibilities:

(a) $\left(\frac{2}{3}, -1\right)$

(b) $(14, 17)$

(c) $\left(-\frac{2}{3}, -1\right)$

(d) $\left(\frac{2}{3}, 17\right)$

(e) $(14, -1)$

$4 = 3x+2$

$2 = 3x$

$\frac{2}{3} = x$

New input

$g\left(\frac{2}{3}\right) = 9 + f\left(3 \cdot \frac{2}{3} + 2\right)$

$g\left(\frac{2}{3}\right) = 9 + f(4)$

$g\left(\frac{2}{3}\right) = 9 + 8$

$g\left(\frac{2}{3}\right) = 17 \Longrightarrow \boxed{\left(\frac{2}{3}, 17\right)}$

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12. Which sequence of transformations will transform the graph of the function f into the graph of the function g?

$$f(x) = \sqrt{x} + 4 \qquad g(x) = \sqrt{\underbrace{x-3}} \underbrace{+6}$$

*affect to output

substitute new input

*affect to INPUT

Possibilities:

(a) shift right by 3 then shift down by 2

(b) shift left by 3 then shift up by 2

(c) shift left by 2 then shift down by 3

(d) shift right by 3 then shift up by 2

(e) shift left by 3 then shift down by 2

$f(x-3) = \sqrt{x-3} + 4$ need this to be "6"

$f(x-3)+2 = \left(\sqrt{\underline{x-3}} + 4\right) \underline{+2}$

*adjustment to INPUT

adjustment to OUTPUT

$f(x-3)+2 = \underbrace{\sqrt{x-3} + 6}_{g(x)}$

$\boxed{\text{shifts graph RIGHT 3 units}}$

$\boxed{\text{shifts graph UP 2 units}}$

13. Use algebra to find the inverse of the given one-to-one function.

*to find inverse function $f(x) = (x^5+9)^4$

→ 1. set equal to "y" → $y = (x^5+9)^4$

→ 2. switch variables → $x = (y^5+9)^4$

Possibilities: → 3. solve for "y"

(a) $f^{-1}(x) = \sqrt[5]{\sqrt[4]{x} - 9}$

(b) $f^{-1}(x) = \sqrt[5]{\sqrt[9]{x} - 4}$

(c) $f^{-1}(x) = x^{20} + 9$

(d) $f^{-1}(x) = \sqrt[4]{\sqrt[5]{x} - 9}$

(e) $f^{-1}(x) = (x^4+9)^5$

$\sqrt[4]{x} = y^5 + 9$ ⟸ NOTE: Technically, should be $\pm\sqrt[4]{x}$, but that is NOT 1-1. So, we'll assume just positive root for this solution.

$\sqrt[4]{x} - 9 = y^5$

$\sqrt[5]{\sqrt[4]{x} - 9} = y$

$\sqrt[5]{\sqrt[4]{x} - 9} = f^{-1}(x)$

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14. Use algebra to find the inverse of the given one-to-one function. $f(x) = \dfrac{4x}{8x+9}$

1. equal to y
2. switch variables
3. solve for y

Possibilities:

(a) $f^{-1}(x) = \frac{1}{2}x + 9$

(b) $f^{-1}(x) = \dfrac{4x}{8x-9}$

(c) $f^{-1}(x) = \dfrac{9x}{4-8x}$

(d) $f^{-1}(x) = \dfrac{8x+9}{4x}$

(e) $f^{-1}(x) = \dfrac{9x}{4x+8}$

$y = \dfrac{4x}{8x+9}$

$x = \dfrac{4y}{8y+9}$ clear denominator

$x(8y+9) = 4y$ distribute x

$8xy + 9x = 4y$ get y values together

$8xy - 4y = -9x$ factor out y

$y(8x-4) = -9x$ divide

$y = \dfrac{-9x}{8x-4}$ factor out − of denominator

$y = \dfrac{-9x}{-(4-8x)}$ simplify

$\boxed{y = \dfrac{9x}{4-8x}}$

15. Write the given expression without using radicals. ← *property of exponents

$$\sqrt[7]{x^5}$$

write root as exponent

$(x^5)^{1/7}$

multiply exponents

$x^{5 \cdot \frac{1}{7}}$

$\boxed{x^{\frac{5}{7}}}$

$x^{a/b} = \sqrt[b]{x^a}$

$(x^a)^{1/b} = \sqrt[b]{x^a}$

Possibilities:

(a) $x^5 - x^7$

(b) x^2

(c) x^{-2}

(d) $x^{7/5}$

(e) $x^{5/7}$

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16. A weekly census of the tree-frog population in Frog Hollow State Park produces the following results.

Week: 0	1	2	3	4	5	6
Frogs: 15	45	135	405	1215	3645	10935

×3 ×3 ×3 ×3 ×3 ×3

Which exponential growth model most closely matches the observations, if t is the week number?

Possibilities:

(a) $15\left(9^{(t/7)}\right)$

(b) $3\left(45^t\right)$

(c) $15\left(3^t\right)$

(d) $3\left(45^{(t/7)}\right)$

(e) $45\left(9^t\right)$

$P(t) = P_0 a^t$

*exponential growth model where $a > 1$

*need P_0 & a

can observe pattern to see that at week 0, population must have been $\frac{45}{3} = 15$

use any data point to solve for "a"

$P(t) = 15a^t$

$P(1) = 15a^1$

$45 = 15a$

$3 = a$

$\boxed{P(t) = 15(3)^t}$

17. Determine how much money (to the nearest cent) will be in a savings account if the initial deposit was \$2000 and the interest rate is 3.250% compounded continuously for 7 years.

formula: $P(t) = P_0 e^{rt}$

Possibilities:

(a) \$2510.82

(b) \$2510.85

(c) \$2510.88

(d) \$2510.91

(e) \$2510.94

$P_0 = \$2000$

$r = .0325$

$t = 7$

input in formula

$P(t) = 2000e^{.0325t}$

raise "e" to exponent 1st $\rightarrow$ $P(7) = 2000e^{(.0325)(7)}$

$P(7) = 2000e^{.2275}$

multiply by 2000 last $\rightarrow$ $P(7) = \$2510.91$

round to dollars & cents (2 decimal places)

18. Translate the given exponential statement into an equivalent logarithmic statement.

$$4^x = 8$$

*Definition of log function:

$a^x = y$ if and only if $\log_a(y) = x$

$a=4$, $y=8$

$4^x = 8 \Longleftrightarrow \boxed{\log_4(8) = x}$

Possibilities:

(a) $\log_4(8) = x$

(b) $\log_8(4) = x$

(c) $\log_8(x) = 4$

(d) $\log_x(4) = 8$

(e) $\log_4(x) = 8$

19. Write the domain of the function $h(x) = \log(x-3)$ in interval notation.

log functions require input values that are strictly positive!

Possibilities:

(a) $(-\infty, 3) \cup (3, \infty)$

(b) $(-\infty, -3)$

(c) $(-\infty, 3]$

(d) $(-\infty, \infty)$

(e) $(3, \infty)$

$x - 3 > 0$ ← set input expression > 0 and solve inequality

$x > 3$

$(3, \infty)$ ← write solution set as an interval

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20. Write the given expression as a single logarithm.

$$4\log(x) + \log(8y) - \log(9z)$$

* log property: $\log_a(x^k) = k \cdot \log_a(x)$

rewrite coefficient as exponent

$\underbrace{\log(x)^4 + \log(8y)}_{\text{rewrite sum as product}} - \log(9z)$

*log property: $\log_a(xy) = \log_a(x) + \log_a(y)$

$\underbrace{\log[x^4(8y)] - \log(9z)}$

$\log\left[\dfrac{x^4 \cdot 8y}{9z}\right]$ ← rewrite difference as quotient

* log property: $\log_a\left(\dfrac{x}{y}\right) = \log_a(x) - \log_a(y)$

Possibilities:

(a) $\log\left(\dfrac{x^4(8y)}{9z}\right)$

(b) $\log\left(x^4 y^8 z^9\right)$

(c) $\log\left(\dfrac{x^4 y^8}{z^9}\right)$

(d) $\log(4x + 8y - 9z)$

(e) $\log(4x(8+y) - 9 - z)$

$\boxed{\text{Formula Sheet:}}$

Compound Interest: If a principal P_0 is invested at an interest rate r for a period of t years, then the amount $P(t)$ of the investment is given by:

$$P(t) = P_0\left(1 + \frac{r}{n}\right)^{nt} \quad \text{(if compounded } n \text{ times per year)}$$

$$P(t) = P_0\, e^{rt} \quad \text{(if compounded continuously).}$$

Change of Base Formula: Let a and b be two positive numbers with $a, b \neq 1$. If $x > 0$, then:

$$\boxed{\log_a(x) = \frac{\log_b(x)}{\log_b(a)}}$$