MA109 — College Algebra Spring 2016
Final Exam 2016-05-03

Name: Solutions **Sec.:** _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1.	a	**b**	c	d	e	12.	a	b	c	**d**	e
2.	**a**	b	c	d	e	13.	a	**b**	c	d	e
3.	a	b	**c**	d	e	14.	a	**b**	c	d	e
4.	a	b	**c**	d	e	15.	**a**	b	c	d	e
5.	**a**	b	c	d	e	16.	a	b	**c**	d	e
6.	a	b	c	**d**	e	17.	a	b	**c**	d	e
7.	a	**b**	c	d	e	18.	a	b	c	d	**e**
8.	a	b	c	d	**e**	19.	a	b	c	**d**	e
9.	a	b	c	d	**e**	20.	**a**	b	c	d	e
10.	a	b	c	**d**	e	21.	a	**b**	c	d	e
11.	a	b	c	d	**e**	22.	a	b	**c**	d	e

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Name: _____

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Solve for r in:

$$(r^2 - 49)(4r - 8) = 0$$

Possibilities:

(a) The only real solutions are ± 7.
(b) The only real solutions are 2 and ± 7.
(c) The only real solutions are 4 and 49.
(d) The only real solutions are 8 and 49.
(e) The only real solutions are 4 and 0.

$r^2 - 49 = 0$ or $4r - 8 = 0$
$\Rightarrow r^2 = 49$ or $r = \frac{8}{4}$
$\Rightarrow r = \pm 7$ or $r = 2$

2. The point $(7, 4)$ is on the graph of which of the following equations?

Possibilities:

(a) $4x + 28 = 28 + xy$
(b) $xy = 0$
(c) $4x + 28 = 4y + 28$
(d) $28 + xy = 16 + xy$
(e) $x = y - 3$

$4(7) + 28 = 28 + (7)(4)$? $\Rightarrow 28 + 28 = 28 + 28$? Yes!
$4(7) = 0$? No
$4(7) + 28 = 4(4) + 28$? $\Rightarrow 28 + 28 = 16 + 28$? No
$28 + 4(7) = 16 + 4(7)$? $28 = 16$? No
$7 = 4 - 3$? No

3. Let

$$f(x) = \begin{cases} 3x - 1 & \text{if } x \leq -2 \\ x^2 + 3 & \text{if } -2 < x \leq 5 \\ -2x - 5 & \text{if } x > 5 \end{cases}$$

Find $f(4)$.

$x = 4 \Rightarrow -2 < 4 \leq 5 \Rightarrow f(4) = 4^2 + 3 = 19$

Possibilities:

(a) 11
(b) -13
(c) 19
(d) 209
(e) 4

4. Solve for z.

$$2z^2 - 9z + 3 = 0$$

$a = 2,\ b = -9,\ c = 3$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2(a)}$$

$$= \frac{9 \pm \sqrt{81 - 4(2)(3)}}{2(2)}$$

$$= \frac{9 \pm \sqrt{81 - 24}}{4}$$

$$= \frac{9 \pm \sqrt{57}}{4}$$

Possibilities:

(a) $\frac{9 \pm \sqrt{105}}{4}$

(b) $\frac{-9 \pm \sqrt{57}}{4}$

(c) $\frac{9 \pm \sqrt{57}}{4}$

(d) $\frac{9}{4} \pm \sqrt{75}$

(e) $\frac{-9 \pm \sqrt{105}}{4}$

---

5. Solve the equation.

$$\log(x-4) + \log(x-6) = \log(6x-39)$$

$$\Rightarrow \log((x-4)(x-6)) = \log(6x-39)$$

$$\Rightarrow (x-4)(x-6) = 6x - 39$$

$$\Rightarrow x^2 - 10x + 24 = 6x - 39$$

$$\Rightarrow x^2 - 16x + 63 = 0$$

$$\Rightarrow (x-9)(x-7) = 0$$

$$\Rightarrow x = 9 \text{ and } x = 7$$

Possibilities:

(a) $x = 7$ and $x = 9$

(b) $x = -3$ and $x = -5$

(c) $x = 4$ and $x = 6$

(d) $x = \frac{29}{4}$ only

(e) $x = -6$ and $x = -8$

---

6. Let $f(x) = 4^x$. Which of the following is $f(2)$?

$$f(2) = 4^2 = 16$$

Possibilities:

(a) $\frac{1}{4}$

(b) 3

(c) 2

(d) 16

(e) $\frac{1}{16}$

---

7. The radioactive element Sodium-24 has a half-life of 14.96 hours and is used to measure radiation dose and circulation in human blood. How long should it take for 76 milligrams to decay to 18 milligrams?

Possibilities:

(a) About 63.16 hours

(b) About 31.09 hours

(c) About 867.68 hours

(d) About 0.14 hours

(e) About 2.08 hours

A_0 = initial amount

$A(t)$ = amount at time t in hours

λ = radioactive decay constant

half life = 14.96 $\Leftrightarrow \frac{1}{2}A_0 = A_0 e^{-\lambda(14.96)}$

$\Leftrightarrow \frac{1}{2} = e^{-\lambda(14.96)}$

$\Leftrightarrow \ln(\frac{1}{2}) = -\lambda(14.96)$

$\Leftrightarrow \lambda = -\frac{\ln(\frac{1}{2})}{14.96} \approx 0.0463$

$A(t) = A_0 e^{-\lambda t}$

$\Rightarrow 18 = 76e^{-0.0463t}$

$\Rightarrow t = -\frac{\ln(\frac{18}{76})}{-0.0463}$

$\Rightarrow t \approx 31.09$ hours

8. Find an equation for the line through the points $(6, 3)$ and $(9, 4)$.

Possibilities:

(a) $y + 3 = 3(x + 6)$

(b) $y + 3 = \frac{1}{3}(x + 6)$

(c) $y - 3 = 3(x - 6)$

(d) $y = -3(x - 6) - 3$

(e) $y - 3 = \frac{1}{3}(x - 6)$

x_1, y_1 $\quad x_2, y_2$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-3}{9-6} = \frac{1}{3}$

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 3 = \frac{1}{3}(x - 6)$

9. Which of the following statements best describes the system of equations?

$$\begin{cases} x + y = 4 \\ 2x + 2y = 8 \end{cases}$$

$\Rightarrow -2x + -2y = -2(4)$

$+ \; 2x + 2y = 8$

$0x + 0y = 0$

$\Rightarrow 0 = 0$ always true

the system is dependent

It has an infinite number of solutions.

Possibilities:

(a) The system is consistent. It has exactly one solution which is $(4, 8)$.

(b) The system is inconsistent. Therefore the system has no solutions.

(c) The system is dependent. Two solutions to the system are $(4, 8)$ and $(2, 2)$. One point that is NOT a solution to the system is $(1, 1)$.

$(4,8)$ is not a solution to $x + y = 4$

(d) The system is dependent. Every point is a solution to the system.

(e) The system is dependent. Two solutions to the system are $(1, 3)$ and $(2, 2)$. One point that is NOT a solution to the system is $(0, 0)$.

10. You have already invested \$300 in a stock with an annual return of 3%. How much of an additional \$1,250 should be invested at 4% and how much at 1% so that the total return on the entire \$1,550 is 2%?

The multiple choice problem only asks for the amount at 4%.

Possibilities:

(a) \$933.33 at 4%

(b) \$600 at 4%

(c) \$1,033.33 at 4%

(d) \$316.67 at 4%

(e) \$516.67 at 4%

x = amount invested at 4%
y = amount invested at 1%
$x + y = 1{,}250 \Rightarrow y = 1{,}250 - x$

$0.03(300) + 0.04x + 0.01y = 0.02(1{,}550)$
$\Rightarrow 9 + 0.04x + 0.01(1{,}250 - x) = 31$
$\Rightarrow 9 + 0.04x + 0.01(1{,}250) - 0.01x = 31$
$\Rightarrow 9 + 0.03x + 12.5 = 31$
$\Rightarrow 0.03x = 9.5 \Rightarrow x \simeq 316.67$

---

11. Let $f(x) = 7x^2 + 4x - 18$. Find $\dfrac{f(x+h) - f(x)}{h}$ and simplify. (Assume $h \neq 0$.)

Possibilities:

(a) $14x + 7h$

(b) $\dfrac{6xh + 3h^2 - 2}{h}$

(c) 1

(d) $7h$

(e) $14x + 4 + 7h$

$f(x+h) = 7(x+h)^2 + 4(x+h) - 18$ $\qquad -f(x) = -7x^2 - 4x + 18$
$= 7(x^2 + 2xh + h^2) + 4x + 4h - 18$
$= 7x^2 + 14xh + 7h^2 + 4x + 4h - 18$

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{7x^2 + 14xh + 7h^2 + 4x + 4h - 18 - 7x^2 - 4x + 18}{h}$

$= \dfrac{14xh + 7h^2 + 4h}{h} = \dfrac{h(14x + 7h - 4)}{h} = 14x + 7h + 4$

---

12. Write the domain of the function $h(x) = \log(x - 7)$ in interval notation.

Possibilities:

(a) $(-\infty, 7]$

(b) $(-\infty, \infty)$

(c) $(-\infty, 7) \cup (7, \infty)$

(d) $(7, \infty)$

(e) $(-\infty, -7)$

$x - 7 > 0 \Rightarrow x > 7 \Rightarrow$  $\Rightarrow (7, \infty)$

The next three problems refer to the same function.

$$\mathbf{P(x) = 9 + 6x - x^3}$$

---

13. Which of the following is a factor of $P(x)$? (*See the top of the page.*)

Possibilities:

(a) $(x-4)$
(b) $(x-3)$
(c) $(x-5)$
(d) $(x-1)$
(e) $(x-2)$

Fact: $(x-a)$ is a factor of a polynomial $P(x)$ if and only if $P(a)=0$

$P(4) = 9+6(4)-4^3 = 9+24-64 \neq 0 \Rightarrow (x-4)$ is not a factor of $P(x)$

$P(3) = 9+6(3)-3^3 = 9+18-27 = 0 \Rightarrow (x-3)$ is a factor of $P(x)$

$P(5) = 9+6(5)-5^3 = 9+35-125 \neq 0$

$P(1) = 9+6(1)-1^3 = 15-1 \neq 0$

$P(2) = 9+6(2)-2^3 = 9+12-8 \neq 0$

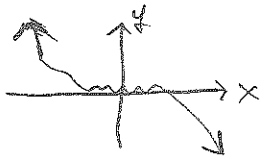
---

14. Determine the end behavior of the graph of $y = P(x)$. (*See the top of the page.*)

Possibilities:

(a) $y \to \infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$
(b) $y \to -\infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$
(c) $y \to 0$ as $x \to \infty$ and $y \to 0$ as $x \to -\infty$
(d) $y \to \infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$
(e) $y \to -\infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$

Leading term of $P(x)$ is $-x^3$
Lead coef $= -1$ (negative)
Degree of $P(x) = 3$ (odd)
End behavior



$y \to \infty$ as $x \to -\infty$ $\quad y \to -\infty$ as $x \to \infty$

---

15. Find the remainder of the division problem $\dfrac{P(x)}{x+3}$. (*See the top of the page.*)

Possibilities:

(a) 18
(b) -250
(c) $74x - 28$
(d) $x^2 - 1$
(e) 194

$$x+3 \overline{)\, -x^3+0x^2+6x+9}$$

$-x^2+3x-3$

$-(-x^3-3x^2)$

$3x^2+6x$

$-(3x^2+9x)$

$-3x+9$

$-(-3x-9)$

18

$\frac{-x^3}{x} = -x^2$

$\frac{3x^2}{x} = 3x$

$\frac{-3x}{x} = -3$

---

16. Suppose the graph of $y = f(x)$ is a parabola with vertex $(1, 3)$ and goes through the point $(0, 5)$. Which of the following is an formula for $f(x)$?

Possibilities:

(a) $f(x) = (x+1)^2 + 5$

(b) $f(x) = (x-1)(x-3)$

(c) $f(x) = 2(x-1)^2 + 3$

(d) $f(x) = 4(x+1)^2 + 3$

(e) $f(x) = (x+1)(x+5)$

formula for a parabola: $f(x) = a(x-h)^2 + k$
(h,k) is the vertex

$f(x) = a(x-1)^2 + 3$

$\Rightarrow 5 = f(0) = a(0-1)^2 + 3 = a(-1)^2 + 3 = a + 3$

$\Rightarrow 5 = a + 3 \Rightarrow a = 2$

$\Rightarrow f(x) = 2(x-1)^2 + 3$

17. Solve for t in $\dfrac{(7t-5)^3}{3} = 9$.

Possibilities:

(a) $5 \pm \sqrt{27}/7$

(b) 2

(c) $\dfrac{8}{7}$

(d) $\dfrac{195112}{3}$

(e) $\dfrac{7}{8}$

$\Rightarrow (7t-5)^3 = 27$

$\Rightarrow 7t - 5 = \sqrt[3]{27} = 3$

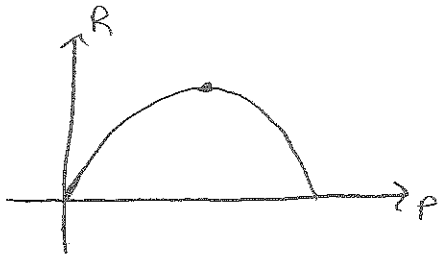
$\Rightarrow 7t = 3 + 5 = 8$

$\Rightarrow t = 8/7$

18. When a high school basketball team charges p dollars per ticket, the total revenue R from ticket sales is given by the formula

$$R(p) = p(1800 - 100p).$$

What per-ticket price maximizes the teams total revenue?



$R(p) = 1800p - 100p^2$

upside down parabola

max is at vertex

Vertex is at $p = \frac{-b}{2a} = -\frac{1800}{2(-100)} = \frac{18}{2} = 9$

Possibilities:

(a) \$0.03

(b) \$18.00

(c) \$49.92

(d) \$8100.00

(e) \$9.00

19. Let $r(x) = \dfrac{7x - 10}{x - 4}$. Find the asymptotes of r.

vertical asymptotes: roots of denominator

horizontal asymptotes: If deg. of numerator = deg of denominator (like in this case) the HA is lead coef. of numer. divided by lead coef. of denom. $= \frac{7}{1} = 7$

Possibilities:

(a) The vertical asymptote is $x = 10$ and the horizontal asymptote is $y = 4$.

(b) The vertical asymptote is $x = 4$ and the horizontal asymptote is $y = 1$.

(c) The vertical asymptote is $x = \frac{10}{7}$ and the horizontal asymptote is $y = 4$.

(d) The vertical asymptote is $x = 4$ and the horizontal asymptote is $y = 7$.

(e) The vertical asymptote is $x = 10$ and the horizontal asymptote is $y = 7$.

20. Explain how the graph of $g(x) = \sqrt{x - 6} + 9$ is obtained from the graph of $f(x) = \sqrt{x}$.

$f(x) = \sqrt{x} \Rightarrow g(x) = \sqrt{x-6} + 9 = f(x-6) + 9$

shift right 6; shift up 9

Possibilities:

(a) Shift the graph of f right 6 units and shift up 9 units to obtain the graph of g.

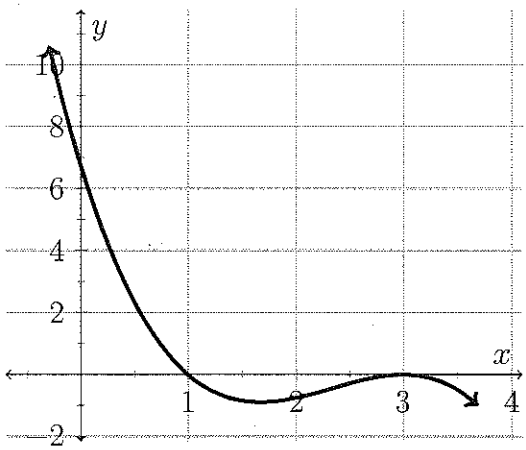
(b) Shift the graph of f left 6 units and shift up 9 units to obtain the graph of g.

(c) Shift the graph of f right 9 units and shift up 6 units to obtain the graph of g.

(d) Shift the graph of f left 9 units and shift down 6 units to obtain the graph of g.

(e) Shift the graph of f right 6 units and shift down 9 units to obtain the graph of g.

The next two problems refer to the graph shown. In the picture below, the graph of the polynomial function $P(x)$ is shown.



---

21. For the graph of the polynomial $P(x)$ drawn above, which of the following can you conclude about P?

Possibilities:

(a) The degree of the polynomial is even and the leading coefficient is positive. ← false. would look like ↑↑

(b) The degree of the polynomial is odd and the leading coefficient is negative.

(c) The parity (even or odd) of the degree of the polynomial or the sign of the leading coefficient can not be determined by the graph. ← false. We can tell the parity by the end behavior.

(d) The degree of the polynomial is even and the leading coefficient is negative. ← false. would look like ↑↓

(e) The degree of the polynomial is odd and the leading coefficient is positive. ← false. would look like ↓↑

---

22. For the graph of the polynomial $P(x)$ drawn above, which of the following statements can be concluded?

(I). $(x-1)$ is a factor of $P(x)$ ← true. The graph of $P(x)$ hits the x-axis at $x=1$

(II). When $P(x)$ is divided by $(x-2)$ the remainder is six. ← false. This would imply $P(2)=6$ but $P(2) \approx -0.6$

(III). $x=1$ is a root with even multiplicity. ← false. $P(x)$ crosses the x-axis at $x=1$, it does not lie tangent to the x-axis at $x=1$

Possibilities:

(a) Only statements (I) and (II) are true.

(b) Statements (I), (II), and (III) are all true.

(c) Only statement (I) is true.

(d) None of the statements are true.

(e) Only statement (III) is true.
