Exam 4

Solutions

Multiple Choice Questions

Form A

1. B	3. A	5. D	7. D	9. A
2. E	4. A	6. C	8. B	10. C

Form B

1. B	3. D	5. D	7. B	9. B
2. E	4. B	6. E	8. A	10. D

Form C

1. A	3. A	5. D	7. C	9. A
2. D	4. D	6. C	8. C	10. D

Free Response Questions

11. (5 points) Find all solutions of $\sin 2x - \cos x = 0$ in the interval $[0, 2\pi)$.

Solution:

$$\begin{aligned} \sin 2x - \cos x &= 0 \\ 2\sin x \cos x - \cos x &= 0 \\ \cos x(2\sin x - 1) &= 0 \\ x &= \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2} \end{aligned}$$

12. (5 points) Given that the terminal point for angle θ is $(20, -21)$, find (must find at least 5 of 6 for all points, otherwise 1 point each)

(a) $\sin\theta$

Solution: $r = \sqrt{20^2 + 21^2} = \sqrt{841} = 29$, so $\sin\theta = -\dfrac{21}{29}$

(b) $\cos\theta$

Solution: $\cos\theta = \dfrac{20}{29}$

(c) $\tan\theta$

Solution: $\tan\theta = -\dfrac{21}{20}$

(d) $\cot\theta$

Solution: $\cot\theta = -\dfrac{20}{21}$

(e) $\sec\theta$

Solution: $\sec\theta = \dfrac{29}{20}$

(f) $\csc\theta$

Solution: $\csc\theta = -\dfrac{29}{21}$

13. Solve the triangle ΔABC given that $c = 30$, $\angle A = 52^\circ$ and $\angle B = 70^\circ$.

(a) (2 points) Find a

Solution: $\dfrac{c}{\sin C} = \dfrac{a}{\sin A}$ so $a = c\dfrac{\sin A}{\sin C} = 30 \sin 52^\circ \sin 58^\circ = 27.876$

(b) (2 points) Find b

Solution: $\dfrac{c}{\sin C} = \dfrac{b}{\sin B}$ so $b = c\dfrac{\sin B}{\sin C} = 30 \sin 70^\circ \sin 58^\circ = 33.242$

(c) (1 point) Find $\angle C$

Solution: $\angle C = 180^\circ - (52^\circ + 70^\circ) = 58^\circ$.

14. Solve the triangle ΔABC given that $b = 15$, $c = 18$ and $\angle A = 108^\circ$.

(a) (3 points) Find a

Solution: Law of Cosines
$a^2 = b^2 + c^2 - 2bc \cos A = 225 + 324 - 2(15)(18) \cos 108^\circ = 715.869$, $a = 26.756$.

(b) (1 point) Find $\angle B$

Solution: $\dfrac{\sin B}{b} = \dfrac{\sin A}{a}$ so $\sin B = \sin A \dfrac{b}{a} = 0.533188$ and $\angle B = 32.22^\circ$

(c) (1 point) Find $\angle C$

Solution: $\angle C = 180^\circ - (\angle A + \angle B) = 39.7788^\circ$

15. A triangular field has sides of length 22, 36, and 44 yards.

(a) (3 points) Find the area.

Solution: By Heron's Formula, $s = \frac{1}{2}(22 + 36 + 44) = 51$

$$\begin{aligned} K &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{51(51-22)(51-36)(51-44)} \\ &= \sqrt{51 \times 29 \times 15 \times 7} \\ &= \sqrt{155295} \\ &= 394.075 \end{aligned}$$

(b) (2 points) Find the largest angle in the triangle.

Solution: The largest angle occurs opposite the largest side so this will be angle opposite 44. By the Law of Cosines

$$\cos A = \frac{22^2 + 36^2 - 44^2}{2 \times 22 \times 36} = -0.09848$$

and $\angle A = 95.6519$.

16. Let $f(x) = 2x^2 + ax + 18$.

(a) (1 point) If $a = 10$ how many solutions are there to the equation $f(x) = 0$. Find them if possible.

> **Solution:** The discriminant is $b^2 - 4ac = a^2 - 144$. If $a = 10$ the discriminant is negative and there are no solutions.

(b) (2 points) If $a = 12$ how many solutions are there to the equation $f(x) = 0$. Find them if possible.

> **Solution:** The discriminant is $b^2 - 4ac = a^2 - 144$. If $a = 12$ the discriminant is zero and there there is exactly one solution which is $-a/4 = -3$.

(c) (2 points) if $a = 13$ how many solutions are there to the equation $f(x) = 0$. Find them if possible.

> **Solution:** The discriminant is $b^2 - 4ac = a^2 - 144$. If $a = 13$ the discriminant is positive and there are two solutions: $x = \dfrac{-13 \pm 5}{4} = -2$ or $-\dfrac{9}{2}$.

17. The arctic lynx population in Northern Canada is given by the function $L(t) = 6000 + 3500 \sin\left(\dfrac{\pi t}{5} + \dfrac{9\pi}{10}\right)$ where the time t is measured in years since the year 2000.

(a) (2 points) What is the largest number of lynx present in the region at any time?

> **Solution:** The largest number of lynx will be 9500.

(b) (3 points) How much time elapses between occurrences of the largest and smallest lynx population?

> **Solution:** This is half of the period. The period is $\dfrac{2\pi}{\pi/5} = 10$. Thus, there are 5 years between the largest and smallest populations.

18. The motion of a projectile that is fired with an initial velocity of v_0 at an angle θ to the horizon at a height of h_0 above the ground is described by the parametric equations

$$x(t) = (v_0 \cos\theta)t$$
$$y(t) = -16t^2 + (v_0 \sin\theta)t + h_0$$

(a) Baseball A is hit with an initial velocity of 98 feet per second at an angle of 35° at a height of 3.5 feet.

$$x(t) = (98\cos 35^\circ)t$$
$$y(t) = -16t^2 + (98\sin 35^\circ)t + 3.5$$

i. (1 point) How long until the ball hits the ground?

Solution: We must solve $y(t) = 0$ which is a simple quadratic equation. It's solutions are $t = -0.0611998$ and $t = 3.57436$. The first value is not useful. Thus the ball hits the ground after 3.57436 seconds.

ii. (1 point) How far did it travel?

Solution: We need to find $x(3.57436) = 286.939$ feet.

(b) Baseball B is hit with an initial velocity of 118 feet per second at an angle of 30° at a height of 3 feet.

$$x(t) = (118\cos 30^\circ)t$$
$$y(t) = -16t^2 + (118\sin 30^\circ)t + 3$$

i. (1 point) How long until the ball hits the ground?

Solution: We must solve $-16t^2 + (118\sin 30^\circ)t + 3 = 0$. By the quadratic formula, we get that $t = -0.050165$ and $t = 3.73767$. The ball hits the ground in 3.73767 seconds.

ii. (1 point) How far did it travel?

Solution: We must find $x(3.73767) = 381.956$ feet.

(c) (1 point) Which ball traveled farther?

Solution: Ball B travels farther.

19. Given that $\cos A = \frac{60}{61}$ and $\sin B = \frac{8}{17}$, find:

(a) (1 point) $\sin A$

Solution: $\sin^2 A = 1 - \cos^2 A = 1 - \left(\frac{60}{61}\right)^2 = \frac{121}{3721}$. Thus $\sin A = \frac{11}{61}$.

(b) (1 point) $\cos B$

Solution: $\cos^2 B = 1 - \sin^2 B = 1 - \left(\frac{8}{17}\right)^2 = \frac{225}{289}$. Thus $\cos B = \frac{15}{17}$.

(c) (1 point) $\sin(A + B)$

Solution:

$$\begin{aligned}\sin(A+B) &= \sin A \cos B + \cos A \sin B \\ &= \left(\frac{11}{61}\right)\left(\frac{15}{17}\right) + \left(\frac{60}{61}\right)\left(\frac{8}{17}\right) \\ &= \frac{645}{1037} \approx 0.62199.\end{aligned}$$

(d) (1 point) $\cos(A + B)$

Solution:

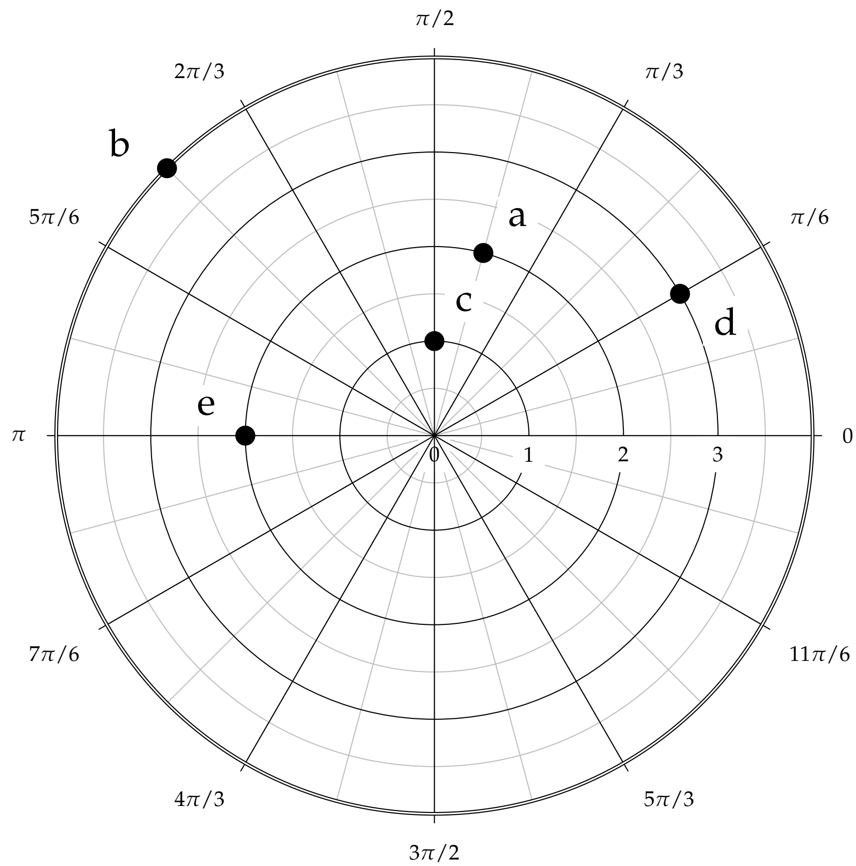
$$\begin{aligned}\cos(A+B) &= \cos A \cos B - \sin A \sin B \\ &= \left(\frac{60}{61}\right)\left(\frac{15}{17}\right) - \left(\frac{11}{61}\right)\left(\frac{8}{17}\right) \\ &= \frac{812}{1037} \approx 0.78303.\end{aligned}$$

(e) (1 point) $\sin 2A$

Solution: $\sin 2A = 2 \sin A \cos A = 2\left(\frac{11}{61}\right)\left(\frac{60}{61}\right) = \frac{1320}{3721} \approx 0.35474$

20. Plot the following points in the attached grid. Label each point with the appropriate letter.

   (a) (1 point) the point with polar coordinates $\left(2, \frac{5\pi}{12}\right)$
   (b) (1 point) the point with polar coordinates $\left(-4, \frac{7\pi}{4}\right)$
   (c) (1 point) the point with polar coordinates $\left(1, \frac{\pi}{2}\right)$
   (d) (1 point) the point with ***rectangular coordinates*** $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$
   (e) (1 point) the point with ***rectangular coordinates*** $(-2, 0)$.



END OF TEST