22   More Trigonometric Identities Worksheet

Concepts:

- Trigonometric Identities

    - Addition and Subtraction Identities
    - Cofunction Identities
    - Double-Angle Identities
    - Power-Reducing Identitites
    - Half-Angle Identities
    - Product-Sum Identities

(**Sections 7.2 & 7.3**)

1. Find the exact values of the following functions using the addition and subtraction formulas

    (a) $\sin \dfrac{9\pi}{12}$

    (b) $\cos \dfrac{7\pi}{12}$

2. Write the expression as the sine or cosine of an angle.

    (a) $\sin \dfrac{\pi}{2} \cos \dfrac{\pi}{7} + \cos \dfrac{\pi}{2} \sin \dfrac{\pi}{7}$

    (b) $\sin 5x \cos x - \cos 5x \sin x$

    (c) $\cos 5x \cos 7x - \sin 5x \sin 7x$

3. Simplify the following expressions as much as possible

    (a) $\tan \left(\dfrac{9\pi}{2} - x \right)$

    (b) $\sin(x + y) - \sin(x - y)$

    (c) $\cos(x + y) - \cos(x - y)$

    (d) $\dfrac{\sin(x + y) - \sin(x - y)}{\cos(x + y) - \cos(x - y)}$

    (e) $\cos \left(x + \dfrac{\pi}{3} \right) + \sin \left(x - \dfrac{\pi}{6} \right)$

4. Verify the following identity:
$$\frac{\cos A - \cos B}{\sin A + \sin B} + \frac{\sin A - \sin B}{\cos A + \cos B} = 0.$$

5. Verify the following identity:
$$\cos(x + y)\cos(x - y) = \cos^2 x - \sin^2 y.$$

6. Use the cofunction identities to evaluate the following expression without using a calculator:
$$\sin^2(21°) + \sin^2(61°) + \sin^2(69°) + \sin^2(29°).$$

7. Find the values of the remaining trigonometric functions of x if

   (a) $\sin x = \dfrac{\sqrt{5}}{3}$ and the terminal point of x is in Quadrant II.

   (b) $\tan x = -\dfrac{\sqrt{11}}{5}$ and $\cos x > 0$.

8. Simplify the expression $\dfrac{\sin 14x}{\sin 13x + \sin x}$.

9. Let $f(x) = \sin 4x + \sin 5x$. Verify that $f(x) = 2\cos\dfrac{x}{2}\sin\dfrac{9}{2}x$.

10. Use an appropriate half-angle formula to find the exact value of each expression.

    (a) $\sin\dfrac{\pi}{12}$

    (b) $\cos\dfrac{\pi}{8}$

    (c) $\tan\dfrac{\pi}{12}$

    (d) $\cos\dfrac{7\pi}{8}$

    (e) $\sin\dfrac{3\pi}{8}$

11. Use an appropriate half-angle formula to simplify $\sqrt{\dfrac{1 - \cos 10x}{2}}$.

12. Use an appropriate power-reducing formula to rewrite $cos^4 x sin^2 x$ in terms of the first power of cosine.

13. Write the product $7\cos 6x \cos 7x$ as a sum.

14. Write the sum $\sin 2x - \sin 7x$ as a product.