

Worksheet 20 - Parametric Equations (§10.5)

In Exercises 1 - 20, plot the set of parametric equations by hand. Be sure to indicate the orientation imparted on the curve by the parametrization.

1.
$$\begin{cases} x = 4t - 3 \\ y = 6t - 2 \end{cases} \text{ for } 0 \leq t \leq 1$$

3.
$$\begin{cases} x = 2t \\ y = t^2 \end{cases} \text{ for } -1 \leq t \leq 2$$

5.
$$\begin{cases} x = t^2 + 2t + 1 \\ y = t + 1 \end{cases} \text{ for } t \leq 1$$

7.
$$\begin{cases} x = t \\ y = t^3 \end{cases} \text{ for } -\infty < t < \infty$$

9.
$$\begin{cases} x = \cos(t) \\ y = \sin(t) \end{cases} \text{ for } -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

11.
$$\begin{cases} x = -1 + 3 \cos(t) \\ y = 4 \sin(t) \end{cases} \text{ for } 0 \leq t \leq 2\pi$$

13.
$$\begin{cases} x = 2 \cos(t) \\ y = \sec(t) \end{cases} \text{ for } 0 \leq t < \frac{\pi}{2}$$

15.
$$\begin{cases} x = \sec(t) \\ y = \tan(t) \end{cases} \text{ for } -\frac{\pi}{2} < t < \frac{\pi}{2}$$

17.
$$\begin{cases} x = \tan(t) \\ y = 2 \sec(t) \end{cases} \text{ for } -\frac{\pi}{2} < t < \frac{\pi}{2}$$

19.
$$\begin{cases} x = \cos(t) \\ y = t \end{cases} \text{ for } 0 \leq t \leq \pi$$

2.
$$\begin{cases} x = 4t - 1 \\ y = 3 - 4t \end{cases} \text{ for } 0 \leq t \leq 1$$

4.
$$\begin{cases} x = t - 1 \\ y = 3 + 2t - t^2 \end{cases} \text{ for } 0 \leq t \leq 3$$

6.
$$\begin{cases} x = \frac{1}{9}(18 - t^2) \\ y = \frac{1}{3}t \end{cases} \text{ for } t \geq -3$$

8.
$$\begin{cases} x = t^3 \\ y = t \end{cases} \text{ for } -\infty < t < \infty$$

10.
$$\begin{cases} x = 3 \cos(t) \\ y = 3 \sin(t) \end{cases} \text{ for } 0 \leq t \leq \pi$$

12.
$$\begin{cases} x = 3 \cos(t) \\ y = 2 \sin(t) + 1 \end{cases} \text{ for } \frac{\pi}{2} \leq t \leq 2\pi$$

14.
$$\begin{cases} x = 2 \tan(t) \\ y = \cot(t) \end{cases} \text{ for } 0 < t < \frac{\pi}{2}$$

16.
$$\begin{cases} x = \sec(t) \\ y = \tan(t) \end{cases} \text{ for } \frac{\pi}{2} < t < \frac{3\pi}{2}$$

18.
$$\begin{cases} x = \tan(t) \\ y = 2 \sec(t) \end{cases} \text{ for } \frac{\pi}{2} < t < \frac{3\pi}{2}$$

20.
$$\begin{cases} x = \sin(t) \\ y = t \end{cases} \text{ for } -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

In Exercises 21 - 24, plot the set of parametric equations using a calculator or computer. Be sure to indicate the orientation imparted on the curve by the parametrization.

21.
$$\begin{cases} x = t^3 - 3t \\ y = t^2 - 4 \end{cases} \text{ for } -2 \leq t \leq 2$$

22.
$$\begin{cases} x = 4 \cos^3(t) \\ y = 4 \sin^3(t) \end{cases} \text{ for } 0 \leq t \leq 2\pi$$

23.
$$\begin{cases} x = e^t + e^{-t} \\ y = e^t - e^{-t} \end{cases} \text{ for } -2 \leq t \leq 2$$

24.
$$\begin{cases} x = \cos(3t) \\ y = \sin(4t) \end{cases} \text{ for } 0 \leq t \leq 2\pi$$

In Exercises 25 - 39, compute parametric equations for the given oriented curve.

25. the directed line segment from $(3, -5)$ to $(-2, 2)$
26. the directed line segment from $(-2, -1)$ to $(3, -4)$
27. the curve $y = 4 - x^2$ from $(-2, 0)$ to $(2, 0)$.
28. the curve $y = 4 - x^2$ from $(-2, 0)$ to $(2, 0)$
(Shift the parameter so $t = 0$ corresponds to $(-2, 0)$.)
29. the curve $x = y^2 - 9$ from $(-5, -2)$ to $(0, 3)$.
30. the curve $x = y^2 - 9$ from $(0, 3)$ to $(-5, -2)$.
(Shift the parameter so $t = 0$ corresponds to $(0, 3)$.)
31. the circle $x^2 + y^2 = 25$, oriented counter-clockwise
32. the circle $(x - 1)^2 + y^2 = 4$, oriented counter-clockwise
33. the circle $x^2 + y^2 - 6y = 0$, oriented counter-clockwise
34. the circle $x^2 + y^2 - 6y = 0$, oriented *clockwise*
(Shift the parameter so t begins at 0.)
35. the circle $(x - 3)^2 + (y + 1)^2 = 117$, oriented counter-clockwise
36. the ellipse $(x - 1)^2 + 9y^2 = 9$, oriented counter-clockwise
37. the ellipse $9x^2 + 4y^2 + 24y = 0$, oriented counter-clockwise
38. the triangle with vertices $(0, 0)$, $(3, 0)$, $(0, 4)$, oriented counter-clockwise
(Shift the parameter so $t = 0$ corresponds to $(0, 0)$.)