Name: ______________________________ Section and/or TA: ______________

Do not remove this answer page. You will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or communication capabilities is permitted. You may not use a phone or other communication device during the exam.

The exam consists of 12 multiple choice questions that are worth 5 points each and 4 free response questions that are worth 10 points each. You should work the multiple choice questions on the question page. After you have checked your work carefully, record your answers by completely filling in the circle below that corresponds to your answer. If you must change your answer, make a note on the front of the exam. Be sure to check carefully when you transfer your answers to the cover sheet.

Show all work to receive full credit on the free response problems. You do not need to compute a decimal approximation to your answer. For example, the answer 4π is preferred to 12.57.

Multiple Choice Questions

1 (A) (B) (C) (D) (E)   **7** (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)   **8** (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)   **9** (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)   **10** (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)   **11** (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)   **12** (A) (B) (C) (D) (E)

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

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Multiple Choice Questions

1. (5 points) Give the domain of the function $f(x) = \dfrac{x^2 - 64}{x^2 + 4x}$.

   A. $(-\infty, -4) \cup (-4, \infty)$

   B. $(-\infty, -4) \cup (-4, 0) \cup (0, \infty)$

   C. $(-\infty, 0) \cup (0, \infty)$

   D. $(-\infty, -8) \cup (-8, 4) \cup (4, \infty)$

   E. $(-\infty, -8) \cup (-8, 0) \cup (0, \infty)$

2. (5 points) Let $f(x) = \dfrac{1}{x^2}$ with the domain $(-\infty, -1]$. Find the inverse function $f^{-1}(x)$.

   A. $f^{-1}(x) = \dfrac{1}{\sqrt{x}}$ with the domain $[1, \infty)$

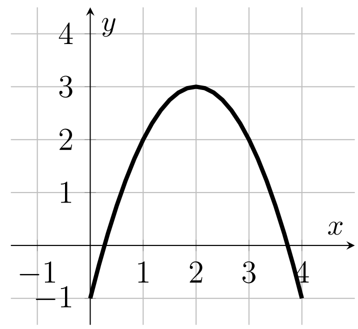
   B. $f^{-1}(x) = \sqrt{x}$ with the domain $[1, \infty)$

   C. $f^{-1}(x) = \dfrac{-1}{\sqrt{x}}$ *with the domain* $(0, 1]$

   D. $f^{-1}(x) = x^2$ with the domain $[1, \infty)$

   E. $f^{-1}(x) = \dfrac{1}{\sqrt{x}}$ with the domain $(-\infty, -1]$

3. (5 points) The graph below gives the position of an object at time x. Find the average velocity over $[0, 4]$.



A. 2 meters/second

B. 0 meters/second

C. -4 meters/second

D. -2 meters/second

E. 4 meters/second

4. (5 points) Consider the function f defined by

$$f(x) = \begin{cases} x+1, & x \leq 1 \\ -x^2+2x-1, & 1 < x \end{cases}.$$

Find the one-sided limit $\lim_{x \to 1^-} f(x)$.

A. 0

B. 1

C. -1

D. 2

E. The limit does not exist

5. (5 points) Suppose $\lim_{x\to 3} f(x) = 8$ and $\lim_{x\to 3} g(x) = 9$. Find

$$\lim_{x\to 3}\left(\frac{(x-1)^3}{f(x)} + (x-2)^2 g(x)\right).$$

A. 39/4

B. -8

C. 19/2

D. 0

E. 10

6. (5 points) Use the Squeeze Theorem to compute the limit $\lim_{x\to 0} x\cos\left(\frac{9}{x}\right)$.

A. 0

B. 9

C. 1/9

D. 1

E. The limit does not exist

7. (5 points) Consider the function f defined by

$$f(x) = \begin{cases} \sqrt{x}, & \text{if } 0 \le x \le 4 \\ -x + a & \text{if } 4 < x \end{cases}.$$

For which value of a is the function f continuous on $[0, \infty)$?

A. $a = -2$

B. $a = 6$

C. $a = 4$

D. $a = 2$

E. This function is not continuous for any value of a.

8. (5 points) Select the statement that best describes the behavior of $f(x) = \dfrac{113}{x}$ near $x = 0$.

A. $\lim_{x \to 0^+} f(x) = -\infty$ and $\lim_{x \to 0^-} f(x) = -\infty$

B. $\lim_{x \to 0^+} f(x) = -\infty$ and $\lim_{x \to 0^-} f(x) = +\infty$

C. $\lim_{x \to 0^+} f(x) = +\infty$ and $\lim_{x \to 0^-} f(x) = +\infty$

D. $\lim_{x \to 0^+} f(x) = +\infty$ *and* $\lim_{x \to 0^-} f(x) = -\infty$

E. $\lim_{x \to 0} f(x) = 113$

9. (5 points) Find the limit $\lim_{x\to\infty} \dfrac{2x+11}{4x-3}$.

   A. 0
   B. $+\infty$
   C. $-\infty$
   D. $-11/3$
   E. $1/2$

10. (5 points) Find the equation of the tangent line to the curve $y = x^2$ at the point $(-1, 1)$.

   A. $y = -\frac{x}{2} + \frac{1}{2}$
   B. $y = -\frac{x}{2} - 1$
   C. $y = 2x - 1$
   D. $y = -2x - 1$
   E. $y = -x$

11. (5 points) Consider the function $f(x) = 5x + 113$. If you evaluate and simplify the expression

$$\frac{f(1+h) - f(1)}{h} \qquad \text{you obtain}$$

A. 5

B. 113

C. 0

D. $5/h$

E. 1

12. (5 points) The graph of f is shown below. For which of the following values of x is the derivative $f'(x) = 0$?

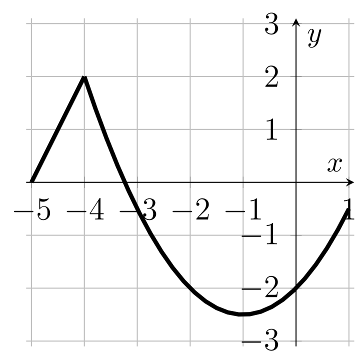
A. $x = -1$

B. $x = -2$

C. $x = 0$

D. $x = -4$

E. $x = -3$



Free response questions: Show all work clearly using proper notation.

13. (10 points) Let $f(x) = \dfrac{x}{6x-4}$.

(a) Find the formula for the inverse function f^{-1}.

(b) Give the domain of f and domain of f^{-1}.

(c) Use the relation between the domain of f and the range of f^{-1} to give the range of f and range of f^{-1}.

Solution: a) Setting $y = \dfrac{x}{6x-4}$, we solve for x. Then we get $y(6x-4) = x$.

Next we get $6xy - 4y = x$. Rearranging terms $6xy - x = 4y$ from which get we $x(6y-1) = 4y$. Finally, $x = \dfrac{4y}{6y-1}$.

Thus, the inverse function f^{-1} is given by

$$f^{-1}(x) = \frac{4x}{6x-1}.$$

b) From the formula defining f, we see that the domain of f is

$$\{x : x \neq \frac{4}{6}\} = (-\infty, 4/6) \cup (4/6, \infty).$$

c) From the formula defining f^{-1}, we see that the domain of f^{-1} is

$$\{x : x \neq \frac{1}{6}\} = (-\infty, 1/6) \cup (1/6, \infty).$$

Since the range of f is domain of f^{-1} and the range of f^{-1} is domain of f, we have

Function	Domain	Range
f	$(-\infty, 4/6) \cup (4/6, \infty)$	$(-\infty, 1/6) \cup (1/6, \infty)$
f^{-1}	$(-\infty, 1/6) \cup (1/6, \infty)$	$(-\infty, 4/6) \cup (4/6, \infty)$

Grading: a) 4 points for solving for x in terms of y , 2 points for the correct formula of f^{-1} in terms of x. (6 total)

b) 1 point for each domain (2 points total).

c) 1 point for each range (2 points total).

Follow through–if the formula for f^{-1} is incorrect, but they give domain of the function they found, then award points in parts b) or c).

Free response questions, show all work, and clearly label your answers

14. (10 points) For each limit, find the limit or state that it does not exist. Show steps clearly using proper notation.

(a) $\displaystyle\lim_{x\to 3} \frac{3x^2-10x+3}{x^2-9}$

(b) $\displaystyle\lim_{x\to -3} \frac{8}{x+3}$

(c) $\displaystyle\lim_{x\to \infty} \frac{3x^2-10x+3}{x^2-9}$

Solution: 14 a)

$$\lim_{x\to 3} \frac{3x^2-10x+3}{x^2-9} = \lim_{x\to 3} \frac{(3x-1)(x-3)}{(x-3)(x+3)} = \lim_{x\to 3} \frac{(3x-1)}{(x+3)} = \frac{3(3)-1}{3+3} = \frac{8}{6}$$

-1 point if drop limit in equalities or keep writing limit after evaluation. (total 4)

14 b) Here we have

$$\lim_{x\to -3} \frac{8}{x+3} = DNE$$

because the right-hand and left-hand limits differ as x approaches -3. Specifically,

$$\lim_{x\to -3^+} \frac{8}{(x+3)} = +\infty$$

$$\lim_{x\to -3^-} \frac{8}{(x+3)} = -\infty$$

The easiest way to see this is to note that the graph of the function

$$y = \frac{8}{x+3}$$

shows a vertical asymptote at $x=-3$. (total 3)

14c) By dividing by the highest power of x in the denominator, we have

$$\lim_{x\to \infty} \frac{3x^2-10x+3}{x^2-9} = \lim_{x\to \infty} \frac{3-10/x+3/x^2}{1-9/x^2} = 3$$

Final answer: 1 point. Justification: 2 points.

Free response questions, show all work, and clearly label your answers

15. (10 points) (a) State the intermediate value theorem.

    (b) Use the intermediate value theorem to show $\frac{1}{x} - 2x = 0$ has a solution. Be sure to give the interval on which you are applying the intermediate value theorem.

Solution: a) Theorem: If f is continuous on the interval $[a, b]$ and Y lies between $f(a)$ and $f(b)$, then there is a value $c \in [a, b]$ so that $f(c) = Y$.

b) Let $f(x) = \frac{1}{x} - 2x$. Since f is a difference of two continuous functions, it is continuous everywhere. We try a few values. For example $f(1) = -1$, $f(2) = -\frac{7}{2}$, and $f(1/2) = 1$. Since $f(1/2) > 0$ and $f(1) < 0$ and f is continuous on the interval $[1/2, 1]$, then there is a number c in $[1/2, 1]$ so that $f(c) = 0$.

Grading: a) Hypotheses (2 points), conclusion (2 points). Give full credit if they use the open interval for $c \in (a, b)$. b) Give interval (1 point), show 0 lies between values at endpoints (3 points), observe function is continuous (2 points).

Free response questions, show all work, and clearly label your answers

16. (10 points) Let $f(x) = \sqrt{x}$. Use the **limit definition** of the derivative to find the derivative $f'(64)$.

Solution: We write the difference quotient and simplify

$$\begin{aligned}
\frac{f(64+h)-f(64)}{h} &= \frac{\sqrt{64+h}-\sqrt{64}}{h} \\
&= \frac{\sqrt{64+h}-\sqrt{64}}{h}\left(\frac{\sqrt{64+h}+\sqrt{64}}{\sqrt{64+h}+\sqrt{64}}\right) \qquad \text{multiply by conjugate} \\
&= \left(\frac{\sqrt{64+h}^2-\sqrt{64}^2}{h}\right)\left(\frac{1}{\sqrt{64+h}+\sqrt{64}}\right), \\
&= \left(\frac{64+h-64}{h}\right)\left(\frac{1}{\sqrt{64+h}+8}\right), \\
&= \left(\frac{h}{h}\right)\left(\frac{1}{8+\sqrt{64+h}}\right), \\
&= \left(\frac{1}{8+\sqrt{64+h}}\right), \qquad \text{if } h \neq 0.
\end{aligned}$$

Thus,

$$f'(64) = \lim_{h\to 0}\frac{f(64+h)-f(64)}{h} = \lim_{h\to 0}\left(\frac{1}{8+\sqrt{64+h}}\right) = \frac{1}{16}.$$

Grading: Form difference quotient (3 points), simplify difference quotient to cancel h (3 points), and find limit (4 points).