Name: ______________________ Student ID: ______________________

Section: ______________________

Do not remove this answer page. You will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or communication capabilities is permitted. You may not use a phone or other communication device during the exam.

The exam consists of 12 multiple choice questions that are worth 5 points each and 4 free response questions that are worth 10 points each. You should work the multiple choice questions on the question page. After you have checked your work carefully, record your answers by completely filling in the circle below that corresponds to your answer. If you must change your answer, make a note on the front of the exam. Be sure to check carefully when you transfer your answers to the cover sheet.

Show all work to receive full credit on the free response problems. You do not need to compute a decimal approximation to your answer. For example, the answer 4π is preferred to 12.57.

Multiple Choice Questions

1	A	B	C	D	E	**7**	A	B	C	D	E
2	A	B	C	D	E	**8**	A	B	C	D	E
3	A	B	C	D	E	**9**	A	B	C	D	E
4	A	B	C	D	E	**10**	A	B	C	D	E
5	A	B	C	D	E	**11**	A	B	C	D	E
6	A	B	C	D	E	**12**	A	B	C	D	E

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

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Multiple Choice Questions

1. (5 points) Give the domain of the function $f(x) = \sqrt{9 - x^2}$.

   A. $[0, 3]$

   B. $(0, \infty)$

   C. $[-3, 3]$

   D. $(9, \infty)$

   E. $(-\infty, 3)$

2. (5 points) Let $f(x) = \dfrac{x+5}{x-2}$. Find the inverse function $f^{-1}(x)$ and give its domain.

   A. $f^{-1}(x) = \dfrac{x-2}{x+5}$ with the domain $(-\infty, 2) \cup (2, \infty)$

   B. $f^{-1}(x) = \dfrac{x+2}{x-5}$ with the domain $(-\infty, 5) \cup (5, \infty)$

   C. $f^{-1}(x) = \dfrac{2x+5}{x-1}$ *with the domain* $(-\infty, 1) \cup (1, \infty)$

   D. $f^{-1}(x) = \dfrac{5x+2}{x+1}$ with the domain $(-\infty, -1) \cup (-1, \infty)$

   E. $f^{-1}(x) = \dfrac{x+2}{x-1}$ with the domain $(-\infty, 1) \cup (1, \infty)$

3. (5 points) The graph below gives the position of an object at time x. Find the average velocity over $[0, 3]$.

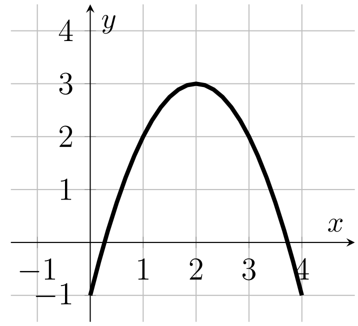
   A. 2 meters/second

   B. 4/3 meters/second

   C. 1 meters/second

   D. 3/4 meters/second

   E. -2 meters/second



4. (5 points) Consider the function f defined by

$$f(x) = \begin{cases} x+1, & x \leq 1 \\ -x^2 + 2x, & 1 < x \end{cases}.$$

Find the one-sided limit $\lim_{x \to 1^+} f(x)$.

   A. 1

   B. 0

   C. -1

   D. 2

   E. The limit does not exist

5. (5 points) Find $\lim_{x \to 32} (31x + 32)(x^2 - 24)$.

   A. 204800

   B. 0

   C. 1000

   D. 1024

   E. 1024000

6. (5 points) Find $\lim_{x \to \pi} \frac{1}{\sin(x)}$.

   A. 1

   B. 1/9

   C. 9

   D. 0

   E. The limit does not exist

7. (5 points) Consider the function f defined by

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2}, & \text{if } x \neq 2 \\ a & \text{if } x = 2 \end{cases}.$$

For which value of a is the function f continuous on $(-\infty, \infty)$?

A. $a = -2$

B. $a = 2$

C. $a = 4$

D. $a = 6$

E. This function is not continuous for any value of a.

8. (5 points) Select the statement that best describes the behavior of $f(x) = \dfrac{1}{2x}$ near $x = 0$.

A. $\lim_{x\to 0^+} f(x) = -\infty$ and $\lim_{x\to 0^-} f(x) = +\infty$

B. $\lim_{x\to 0^+} f(x) = -\infty$ and $\lim_{x\to 0^-} f(x) = -\infty$

C. $\lim_{x\to 0^+} f(x) = +\infty$ *and* $\lim_{x\to 0^-} f(x) = -\infty$

D. $\lim_{x\to 0^+} f(x) = +\infty$ and $\lim_{x\to 0^-} f(x) = +\infty$

E. $\lim_{x\to 0} f(x) = 113$

9. (5 points) Find the limit $\lim_{x\to\infty} \dfrac{x^2+2x+11}{2x^2-4x-3}$.

   A. 1/2

   B. $+\infty$

   C. $-\infty$

   D. 0

   E. 2

10. (5 points) Find the equation of the tangent line to the curve $y = -x^2 + 2x$ at the point $(2, 0)$.

    A. $y = -2x + 2$

    B. $y = 2x - 1$

    C. $y = -\frac{x}{2} + \frac{1}{2}$

    D. $y = -\frac{x}{2} - 1$

    E. $y = -2x + 4$

11. (5 points) Consider the function $f(x) = 113x + 5$. If you evaluate and simplify the expression

$$\frac{f(1+h) - f(1)}{h} \qquad \text{you obtain}$$

A. 1

B. 113

C. 5

D. 0

E. $5/h$

12. (5 points) The graph of f is shown below. Find the value(s) of x for which the derivative $f'(x)$ does not exist.

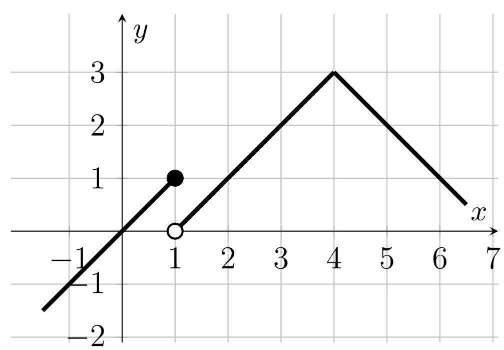
A. $x = 0$ and $x = 2$

B. $x = -1$ and $x = 0$

C. $x = 2$ and $x = 3$

D. $x = 1$ *and* $x = 4$

E. $x = 3$ and $x = 5$



Free response questions: Show all work clearly using proper notation.

13. (10 points) Let $f(x) = \dfrac{1}{x^2}$ with the domain $(-\infty, -1]$.

    (a) Find the formula for the inverse function f^{-1}.

    (b) Give the range of f.

    (c) Use the relation between the domain of f^{-1} and the range of f to give the domain and range of f^{-1}.

Solution: a) Setting $y = \dfrac{1}{x^2}$, we solve for x. Then we get $x^2 = \dfrac{1}{y}$.

Next we get $|\, x \,| = \sqrt{\frac{1}{y}}$. Since $x \leq -1$, $|\, x \,| = -x$. Then $-x = \dfrac{1}{\sqrt{y}}$

Finally, $x = \dfrac{-1}{\sqrt{y}}$.

Thus, the inverse function f^{-1} is given by

$$f^{-1}(x) = \frac{-1}{\sqrt{x}}.$$

b) Since $x \leq -1$, we get $x^2 \geq 1$. Then $0 < y = \dfrac{1}{x^2} \leq 1$. Thus, the range of f is

$$\{y : 0 < y \leq 1\} = (0, 1].$$

c) Since the range of f is domain of f^{-1} and the range of f^{-1} is domain of f, we have

Function	Domain	Range
f	$(-\infty, -1]$	(0,1]
f^{-1}	(0,1]	$(-\infty, -1]$

Grading: a) 3 points for solving for x in terms of y , 2 points for the correct formula of f^{-1} in terms of x. (5 points total)

b) 2 points for the range of f (2 points total).

c) 2 points for domain and 1 point for range (3 points total).

Follow through–if the formula for f^{-1} is incorrect, but they give domain of the function they found, then award points in parts b) or c).

Free response questions, show all work, and clearly label your answers

14. (10 points) For each limit, find the limit or state that it does not exist. Show steps clearly using proper notation.

   (a) $\displaystyle\lim_{x\to 0} \frac{x^2-1}{x^2-x-2}$

   (b) $\displaystyle\lim_{x\to -3} \frac{x^2-1}{x^2-x-2}$

   (c) $\displaystyle\lim_{x\to -\infty} \frac{x^2-1}{x^2-x-2}$

Solution: 14 a)

$$\lim_{x\to 0} \frac{x^2-1}{x^2-x-2} = \frac{(0)^2-1}{(0)^2-0-2} = \frac{-1}{-2} = \frac{1}{2}$$

(total 3 points)

14 b) First, we simplify the expression

$$\frac{x^2-1}{x^2-x-2} = \frac{(x+1)(x-1)}{(x-1)(x+2)} = \frac{(x+1)}{(x+2)} \quad \text{for} \quad x \neq 1.$$

Thus,

$$\lim_{x\to -2} \frac{x^2-1}{x^2-x-2} = \lim_{x\to -2} \frac{x+1}{x+2} = DNE$$

because the right-hand and left-hand limits differ as x approaches -2. Specifically,

$$\lim_{x\to -2^+} \frac{x+1}{x+2} = -\infty$$

$$\lim_{x\to -2^-} \frac{x+1}{x+2} = +\infty$$

The easiest way to see this is to note that the graph of the function

$$y = \frac{x+1}{x+2} = 1 - \frac{1}{x+2}$$

shows a vertical asymptote at $x = -2$. (total 4 points)

14c) By dividing by the highest power of x in the denominator, we have

$$\lim_{x\to -\infty} \frac{x^2-1}{x^2-x-2} = \lim_{x\to -\infty} \frac{1-1/x^2}{1-1/x-2/x^2} = 1$$

Final answer: 1 point. Justification: 2 points. (total 3 points)

Free response questions, show all work, and clearly label your answers

15. (10 points) (a) State the Squeeze Theorem.

(b) Use the Squeeze Theorem to compute the limit $\lim_{x\to 0} x^2 \sin\left(\frac{\pi}{x}\right)$.

Solution: a) Squeeze Theorem: Let f, g, and h be three functions such that

$$f(x) \le g(x) \le h(x)$$

for all x in an open interval (a, b) that contains $c \in (a, b)$. If

$$\lim_{x\to c} f(x) = \lim_{x\to c} h(x) = L, \text{ then } \lim_{x\to c} g(x) = L.$$

b) Since $-1 \le \sin\left(\frac{\pi}{x}\right) \le 1$, we have $-x^2 \le x^2 \sin\left(\frac{\pi}{x}\right) \le x^2$.

Since $\lim_{x\to 0} -x^2 = \lim_{x\to 0} x^2 = 0$, applying the Squeeze Theorem we get that

$$\lim_{x\to 0} x^2 \sin\left(\frac{\pi}{x}\right) = 0.$$

Grading: a) Hypotheses (2 points), conclusion (2 points).

b) Give bounds for the function of interest (2 points), show that extreme functions have limit zero (2 points), conclude that the limit is zero by using the Squeeze Theorem (2 points).

Free response questions, show all work, and clearly label your answers

16. (10 points) Let $f(x) = x^2$. Use the **limit definition** of the derivative to find the derivative $f'(8)$.

Solution: We write the difference quotient and simplify

$$\begin{aligned}
\frac{f(8+h)-f(8)}{h} &= \frac{(8+h)^2-(8)^2}{h} \\
&= \frac{64+h^2+16h-64}{h} \\
&= \frac{h^2+16h}{h}, \\
&= \frac{h(h+16)}{h}, \\
&= h+16, \qquad \text{if } h \neq 0.
\end{aligned}$$

Thus,

$$f'(8) = \lim_{h\to 0} \frac{f(8+h)-f(8)}{h} = \lim_{h\to 0} (h+16) = 16.$$

Grading: Form difference quotient (3 points), simplify difference quotient to cancel h (3 points), and find limit (4 points).