Name: ____________________ Student ID: ____________________

Section: ____________________

Do not remove this answer page. You will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or communication capabilities is permitted. You may not use a phone or other communication device during the exam.

The exam consists of 12 multiple choice questions that are worth 5 points each and 4 free response questions that are worth 10 points each. You should work the multiple choice questions on the question page. After you have checked your work carefully, record your answers by completely filling in the circle below that corresponds to your answer. If you must change your answer, make a note on the front of the exam. Be sure to check carefully when you transfer your answers to the cover sheet.

Show all work to receive full credit on the free response problems. You do not need to compute a decimal approximation to your answer. For example, the answer 4π is preferred to 12.57.

Multiple Choice Questions

1	A	B	C	D	E	**7**	A	B	C	D	E
2	A	B	C	D	E	**8**	A	B	C	D	E
3	A	B	C	D	E	**9**	A	B	C	D	E
4	A	B	C	D	E	**10**	A	B	C	D	E
5	A	B	C	D	E	**11**	A	B	C	D	E
6	A	B	C	D	E	**12**	A	B	C	D	E

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

Trigonometric functions for special angles

θ	$\cos(\theta)$	$\sin(\theta)$	θ	$\cos(\theta)$	$\sin(\theta)$	θ	$\cos(\theta)$	$\sin(\theta)$	θ	$\cos(\theta)$	$\sin(\theta)$
0	1	0	$\frac{\pi}{2}$	0	1	π	-1	0	$\frac{3\pi}{2}$	0	-1
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{2\pi}{3}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{7\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{5\pi}{3}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{3\pi}{4}$	$\frac{-\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{5\pi}{4}$	$\frac{-\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	$\frac{7\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{4\pi}{3}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{11\pi}{6}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$

Multiple Choice Questions

1. (5 points) Let $f(x) = x^2 \ln(x^3)$. Find $f'(x)$.

   A. $2x \ln(x^3) + 3x$

   B. $2x \ln(x^3) + x^2$

   C. $2x + \frac{1}{x^3}$

   D. $2x \ln(x^3) + \frac{1}{x}$

   E. $6x \ln(x^3)$

2. (5 points) Find the derivative $f'(x)$ for $f(x) = \dfrac{1}{1+x^2}$.

   A. $\dfrac{2x}{(1+x^2)^2}$

   B. $\dfrac{1-2x}{1+x^2}$

   C. $\dfrac{-2x}{(1+x^2)^2}$

   D. $\dfrac{1}{(1+x^2)^2}$

   E. $\dfrac{1+x^2+x^3}{(1+x^2)^2}$

3. (5 points) Let $f(x) = \dfrac{\cos(x)}{\sin(x)}$. Find $f'(x)$.

   A. $\dfrac{\sin(x)}{\cos(x)}$

   B. $\dfrac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)}$

   C. $\dfrac{\sin^2(x) + \cos^2(x)}{\sin^2(x)}$

   D. $\dfrac{1}{\sin(x)}$

   E. $\dfrac{\sin(x)\cos(x) + \cos(x)\sin(x)}{\sin^2(x)}$

4. (5 points) Let $f(x) = e^{-x^2}$. Find $f'(x)$.

   A. $2e^{-x^2}$

   B. $-e^{-x^2}$

   C. $-2xe^{-x^2}$

   D. e^{-x^2-1}

   E. $-x^2e^{-x^2-1}$

5. (5 points) Let $f(x) = x^2$, $g(x) = 2x$, and $F(x) = f(g(x))$. Find $F'(2)$.

   A. $F'(2) = 4$

   B. $F'(2) = 8$

   C. $F'(2) = 2$

   D. $F'(2) = 32$

   E. $F'(2) = 16$

6. (5 points) Suppose that the equation of motion for a particle is

$$s(t) = \cos(\pi t), \text{ where } s \text{ is in meters and } t \text{ in seconds.}$$

   Find the acceleration after 1 second.

   A. π

   B. π^2

   C. $-\pi^2$

   D. 0

   E. $-\pi$

7. (5 points) Let $f(x) = \cos(\arcsin(x))$. Rewrite $f(x)$ without trigonometric functions. Hint: set $\theta = \arcsin(x)$ and consider the triangle below.

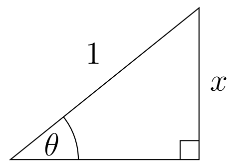
   A. $f(x) = x + 1$

   B. $f(x) = x$

   C. $f(x) = \dfrac{x}{\sqrt{1-x^2}}$

   D. $f(x) = \dfrac{\sqrt{1-x^2}}{x}$

   E. $f(x) = \sqrt{1-x^2}$



8. (5 points) Find the equation of the tangent line to the curve $y = x\cos x$ at the point $(\pi, -\pi)$.

   A. $y = -x + 2\pi$

   B. $y = -x$

   C. $y = x - 2\pi$

   D. $y = x + \pi/2$

   E. $y = -2x$

9. (5 points) Find the smallest, positive value x for which the function $f(x) = x - 2\sin(x)$ has a horizontal tangent line.

   A. $\pi/3$

   B. $\pi/4$

   C. $\pi/2$

   D. $2\pi/3$

   E. $\pi/6$

10. (5 points) Let $f(x) = \cos(\arccos(x))$. Find the derivative of f.

    A. 1

    B. $\dfrac{-x}{\sqrt{1+x^2}}$

    C. $\dfrac{\sqrt{1-x^2}}{x}$

    D. $\dfrac{x}{\sqrt{1+x^2}}$

    E. $\dfrac{-\sqrt{1-x^2}}{x}$

11. (5 points) Consider the curve defined by the equation $x^3 + y^3 = 18xy$. Find the slope of the tangent line at the point $(9, 9)$.

    A. 2

    B. -2

    C. -3

    D. 0

    E. -1

12. (5 points) Let $f(x) = 113\cos(x)$. Find the 32^{nd} derivative $f^{(32)}(x)$.

    A. $113\cos(x)$

    B. $113\cos^{32}(x)$

    C. $-113\cos(x)$

    D. $-113\sin(x)$

    E. $113\sin^{32}(x)$

Free response questions: Show all work clearly using proper notation.

13. (10 points) Consider the curve defined by the equation

$$y = y^3 + xy + x^3.$$

(a) Find the derivative $y' = \dfrac{dy}{dx}$ along the curve.

(b) Find the slope of the tangent line at the point $(1, -1)$.

Solution: a) Differentiating both sides we have

$$y' = 3y^2y' + (y + xy') + 3x^2$$

Solving for the derivative gives

$$y' = \frac{dy}{dx} = \frac{y + 3x^2}{1 - 3y^2 - x}.$$

b) The slope of the tangent line at the point $(1, -1)$ is

$$y'|_{x=1,y=-1} = \frac{-1 + 3(1)^2}{1 - 3(-1)^2 - 1} = \frac{-2}{-3} = \frac{2}{3}.$$

Grading.

a) Differentiate. Chain rule (3 point), product rule (2 point), remaining terms (1 point). Solve for dy/dx (or y') (2 point)

b) Find slope (2 points)

Free response questions: Show all work clearly using proper notation.

14. (10 points)

(a) State the mean value theorem.

(b) Let $f(x) = -(x-1)^2+1$ on the interval $[0,2]$. **Check if the mean value theorem** can be applied to f on $[0,2]$. If so, find all values $c \in [0,2]$ guaranteed by the mean value theorem.

Solution: a) *Theorem.* If f is continuous on a closed interval $[a,b]$ and differentiable on the open interval (a,b), then there is point c in (a,b) so that

$$f'(c) = \frac{f(b)-f(a)}{b-a}.$$

b) f is continuous on $[0,2]$ because it is a polynomial and its derivative $f'(x) = -2(x-1)$ exists for all points in $(0,2)$. Thus, f is differentiable on $(0,2)$. So we can apply the MVT. Thus, by the MVT there is $c \in (0,2)$ such that

$$\frac{f(2)-f(0)}{2-(0)} = f'(c).$$

$$\frac{(-(2-1)^2+1)-(-(0-1)^2+1)}{2-0} = -2(c-1)$$

Simplifying,

$$0 = -2c+2$$

Thus,

$$c = 1.$$

Grading.

a) Continuity, differentiable, intervals (3 points). Existence of c in (a,b) (accept $[a,b]$) (1 point), equation $f'(c) = (f(b)-f(a))/(b-a)$ (1 point).

b) Verify continuity and differentiability hypotheses (1 point), use MVT to obtain $(f(2)-f(0))/2 = f'(c)$ (2 points), find values of c (2 points).

Free response questions: Show all work clearly using proper notation.

15. (10 points) The height (in meters) of a bullet fired in the air vertically from ground level with an initial velocity 113 m/s is

$$s(t) = -4.9t^2 + 113t.$$

(a) Find the velocity and acceleration as functions of t.

(b) Find the velocity after 1 and 2 seconds and give the bullet's maximum velocity.

(c) Find the bullet's maximum height.

Solution: a) Velocity $v(t) = s'(t) = -9.8t + 113$ and accerelation $a(t) = v'(t) = -9.8$.

b) Initial velocity is $v(0) = 113$ m/s. After 1 second the velocity is $v(1) = 113 - 9.8(1) = 103.2$ m/s. After 2 seconds the velocity is $v(2) = 113 - 9.8(2) = 93.4$ m/s. We notice that the velocity is decreasing as time increases. In general, for $t > 0$ we have $v(t) = 113 - 9.8t < 113$. Thus, the initial velocity is the maximum velocity which is equal to 113 m/s.

c) The maximum height will be achieved at time T for which $v(T) = 0$. Thus, we solve $113 - 9.8T = 0$, then $T = 113/9.8 = 11.5306$.

Thus, the maximum height is

$s(T) = -4.9T^2 + 113T = -4.9(11.5306)^2 + 113(11.5306) \approx 651.48$.

Grading.

a) Find $v = s'$ and $a = v'$ (2 points).

b) Noticing that the initial velocity $v(0) = 113$ m/s is the maximum (3 points).

c) Noticing that the maximum height will be achieved when $v(T) = 0$ (2 points).

Finding T such that $v(T) = 0$ (2 points).

Finding maximum height by evaluating $s(T)$ (1 point.)

Free response questions: Show all work clearly using proper notation.

16. (10 points) Let $f(x) = e^{2x}$.
    (a) Find the derivative $f'(x)$.
    (b) Find the equation of the tangent line to the graph of f at $x = 0$.
    (c) Find the equation of the tangent line to the graph of f at $x = 1$.

Solution: The derivative of $f(x) = e^{2x}$ is $f'(x) = 2e^{2x}$. Then $f'(0) = 2$ and $f'(1) = 2e^2$.

a) We find that $f(0) = 1$. Thus, the tangent line at $(0, 1)$ is $y = 2(x-0)+1 = 2x+1$.

b) We find that $f(1) = e^2$. Thus, the tangent line at $(1, e^2)$ is $y = 2e^2(x - 1) + e^2$ which simplifies to $y = 2e^2x - e^2$.

Grading.

a) Derivative (2 points)

b) Line through $(0, 1)$: slope (2 points), equation (2 points)

c) Line through $(1, e^2)$: slope (2 points), equation (2 points)