Name: ______________________ Student ID: ______________________

Section: ______________________

Do not remove this answer page. You will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or communication capabilities is permitted. You may not use a phone or other communication device during the exam.

The exam consists of 12 multiple choice questions that are worth 5 points each and 4 free response questions that are worth 10 points each. You should work the multiple choice questions on the question page. After you have checked your work carefully, record your answers by completely filling in the circle below that corresponds to your answer. If you must change your answer, make a note on the front of the exam. Be sure to check carefully when you transfer your answers to the cover sheet.

Show all work to receive full credit on the free response problems. You do not need to compute a decimal approximation to your answer. For example, the answer 4π is preferred to 12.57.

Multiple Choice Questions

1	A	B	C	D	E	**7**	A	B	C	D	E
2	A	B	C	D	E	**8**	A	B	C	D	E
3	A	B	C	D	E	**9**	A	B	C	D	E
4	A	B	C	D	E	**10**	A	B	C	D	E
5	A	B	C	D	E	**11**	A	B	C	D	E
6	A	B	C	D	E	**12**	A	B	C	D	E

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

Trigonometric functions for special angles

θ	$\cos(\theta)$	$\sin(\theta)$	θ	$\cos(\theta)$	$\sin(\theta)$	θ	$\cos(\theta)$	$\sin(\theta)$	θ	$\cos(\theta)$	$\sin(\theta)$
0	1	0	$\frac{\pi}{2}$	0	1	π	-1	0	$\frac{3\pi}{2}$	0	-1
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{2\pi}{3}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{7\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{5\pi}{3}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{3\pi}{4}$	$\frac{-\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{5\pi}{4}$	$\frac{-\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	$\frac{7\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{4\pi}{3}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{11\pi}{6}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$

Multiple Choice Questions

1. (5 points) Assume that the radius r of a disk is expanding at a rate of 4 inches/min. The area of the disk is given by $A = \pi r^2$.

   Determine the rate at which the area of the disk is changing with respect to time when $r = 3$ inches.

   A. 4π inches^2/min

   B. 16π inches^2/min

   C. 24π inches^2/min

   D. 3π inches^2/min

   E. 12π inches^2/min

2. (5 points) Suppose the population of a shark colony at time t consists of $P(t) = 42e^{0.1t}$ individuals. Find the time it takes for the population to double.

   A. $\ln(2)/0.1$

   B. $42/0.1$

   C. 42

   D. 0.1

   E. $\ln(0.1)/2$

3. (5 points) Let $f(x) = x^4 - 6x^2 + 17x + 3$. Find all the numbers c so that $(c, f(c))$ is an inflection point.

   A. $c = -1$ and $c = 1$

   B. $c = -1$ and $c = 2$

   C. $c = 0$ and $c = 5$

   D. $c = 0$ and $c = 2$

   E. There are no inflection points.

4. (5 points) If possible, find the global maximum and global minimum **values** of $f(x) = x^3 - 3x + 5$ on the interval $[0, 3]$.

   A. global maximum 23; global minimum 3

   B. global maximum 5; global minimum 3

   C. global maximum 23; global minimum 5

   D. global maximum does not exist; global minimum 3

   E. global maximum and global minimum do not exist

5. (5 points) Let $f(x) = e^{2x}$. If possible, find the global maximum and minimum **values** of f on the interval $[1, \infty)$.

   A. Neither the maximum nor the minimum exist.

   B. The maximum is e^2, and the minimum does not exist.

   C. The maximum does not exist, and the minimum is 0.

   D. The maximum is e^2 and the minimum is 0.

   E. The maximum does not exist, and the minimum is e^2.

6. (5 points) Find the intervals where the function

$$f(x) = \frac{x}{x^2 + 1}$$

is increasing and the ones where it is decreasing.

   A. decreasing on $(-\infty, -1)$ and $(1, \infty)$, increasing on $(-1, 1)$

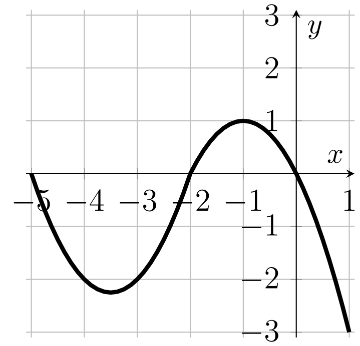
   B. increasing on $(-\infty, -1)$ and $(1, \infty)$, decreasing on $(-1, 1)$

   C. is always increasing

   D. is always decreasing

   E. decreasing on $(-\infty, -1)$ and increasing on $(-1, \infty)$

7. (5 points) The graph below is the **derivative** f' of a function f. Which of the following most accurately describes the behavior of f?



A. decreasing on $(-5, -2)$, decreasing on $(0, 1)$, and increasing on $(-2, 0)$

B. is always decreasing

C. is always increasing

D. decreasing on $(-5, -2)$, increasing on $(0, 1)$, and increasing on $(-2, 0)$

E. increasing on $(-5, -2)$, increasing on $(0, 1)$, and decreasing on $(-2, 0)$

8. (5 points) Let f be a function so that $f''(x) = \frac{4-x^2}{(x^2+1)^2}$. On which interval(s) is f concave down?

A. $(-2, 2)$

B. $(-\infty, \infty)$

C. The function is concave up on $(-\infty, \infty)$

D. $(-\infty, -2)$ and $(2, \infty)$

E. $(2, \infty)$

9. (5 points) Find the value of $\lim_{x \to 0} \dfrac{\sin(42x)}{\ln(1+x)}$.

   A. 42
   B. 0
   C. -42
   D. 1
   E. Does not exist

10. (5 points) Use L'Hôpital's rule to evaluate the following limit

$$\lim_{x \to \infty} \frac{\ln x}{x^2}$$

   A. 1
   B. 1/2
   C. 0
   D. -1
   E. The limit does not exist

11. (5 points) Find the general anti-derivative of $f(x) = 5\sin(x) + 3e^x$.

A. $F(x) = -5\cos(x) + 3e^x + C$

B. $F(x) = -5\cos(x) + e^x + C$

C. $F(x) = 5\sin(x) + 3e^x + C$

D. $F(x) = 5\cos(x) + 3e^x + C$

E. $F(x) = \frac{1}{\sqrt{1-x^2}} + 3e^x + C$

12. (5 points) Find the function f with derivative $f'(x) = \dfrac{5}{x}$ so that $f(e) = 2$.

A. $f(x) = \ln(5x) + 2$

B. $f(x) = x - e + 2$

C. $f(x) = \ln(5x)$

D. $f(x) = 5\ln x - 3$

E. $f(x) = 5\ln x$

Free response questions: Show work clearly with proper notation.

13. (10 points) The half-life of Polonium Po-209 is 102 years. This means that its mass will decrease to half its original value after 102 years. The function $M(t) = M(0)e^{-kt}$ gives the mass of Polonium Po-209 after t years.

    (a) Find the decay rate k.

    (b) Assume that an object initially contains 100 g of Polonium Po-209. Find the time it takes for the mass of Polonium Po-209 to become 20 g.

14. (10 points) A 5 meter ladder leans against a wall. The bottom of the ladder is sliding away from the wall at 0.5 m/sec. When the base of the ladder is 3 meters from the wall, find the speed of the top of the ladder as it slides on the wall.

    (a) Make a sketch summarizing the information in the problem. Label the quantities you use in your solution.

    (b) Find the height of the top of the ladder when the base of the ladder is 3 meters away from the wall.

    (c) Find the speed of the top of the ladder when the base of the ladder is 3 meters from the wall.

15. (10 points) Let $f(x) = (x+5)e^{-x}$. Use calculus to clearly answer the following:

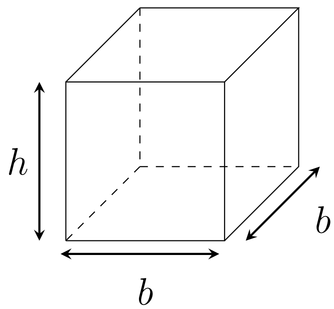
(a) Find the intervals over which $f(x)$ is increasing, and the intervals over which $f(x)$ is decreasing.

(b) Find each value x where f has a local maximum and each value x where f has a local minimum.

(c) Find the intervals over which $f(x)$ is concave up, and the intervals over which $f(x)$ is concave down.

(d) Find all the inflection points.

16. (10 points) An open rectangular box (that is, no top) is constructed out of two different types of metal. The metal for the bottom, which is a square, costs \$6 per square foot. The metal for the sides costs \$3 per square foot. Find the dimensions that minimize cost if the box has a volume of 729 cubic feet. Make sure to justify why your answer is a minimum!



The volume of the box is

$$V = b^2h$$

The cost function is

$$C = 4(3bh) + 6b^2$$