MA 113 Calculus I — Spring 2018

Exam 3 — Tuesday, 10 April 2018

Name: ______________________

Section: ______________________

Last 4 digits of student ID #: ________

This exam has 12 multiple choice questions (five points each) and 4 free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-buds during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box.*

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E

B, C, E, D, A, B, D, D, A, B, E, D

Exam Scores

Question	Score	Total
MC		60
13		10
14		10
15		10
16		10
Total		100

1. The half-life of a radioactive substance is 20 years. A sample of the substance has a mass of 24 grams. In how many years will the substance have a mass of 4 grams? (Remember that $\ln\left(\frac{1}{a}\right) = -\ln(a)$.)

   (A) 60

   (B) $20\frac{\ln(6)}{\ln(2)}$

   (C) 50

   (D) 52

   (E) None of the above

2. Suppose that $\frac{dy}{dt} = ky$, where k is a constant, and suppose that $y(0) = 5$ and $y(2) = 20$. Find $y(6)$.

   (A) 300

   (B) 312

   (C) 320

   (D) 332

   (E) 340

Record the correct answer to the following problems on the front page of this exam.

3. Suppose that $f'(x) = x(x-2)(x-3)$. Find the interval or intervals where f is decreasing. (Read the problem carefully. The given function is $f'(x)$, not $f(x)$.)

   (A) $(3, \infty)$

   (B) $(0, 2)$

   (C) $(-\infty, 0)$ and $(3, \infty)$

   (D) $(0, 2)$ and $(3, \infty)$

   (E) $(-\infty, 0)$ and $(2, 3)$

4. Suppose that $f'(x) = 2x^3 - 9x^2 + 12x$. Find the interval or intervals where the graph of f is concave upward. (Read the problem carefully. The given function is $f'(x)$, not $f(x)$.)

   (A) $(1, \infty)$

   (B) $(-\infty, 2)$

   (C) $(1, 2)$

   (D) $(-\infty, 1)$ and $(2, \infty)$

   (E) None of the above

Record the correct answer to the following problems on the front page of this exam.

5. You are given that $f'(x) = (x-1)(x-2)^2(x-3)(x-4)$. Find the values of x that give the local maximum and local minimum values of the function $f(x)$.

   (A) Local maximum value of f at $x = 3$ and local minimum values of f at $x = 1, 4$

   (B) Local maximum values of f at $x = 1, 4$ and local minimum value of f at $x = 3$

   (C) Local maximum values of f at $x = 1, 3$ and local minimum values of f at $x = 2, 4$

   (D) Local maximum values of f at $x = 2, 4$ and local minimum values of f at $x = 1, 3$

   (E) Cannot be determined from the given information.

6. Find the critical points of $f(x) = \sin(2x)$ in the interval $(0, \pi)$.

   (A) $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$

   (B) $x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$

   (C) $x = \frac{\pi}{2}$

   (D) $x = \frac{\pi}{3}$ and $x = \frac{2\pi}{3}$

   (E) There are no critical points in this interval.

Record the correct answer to the following problems on the front page of this exam.

7. Assume that $f''(x) = x^2(x-1)(x-3)$. Find the points of inflection of the function f.

   (A) $x = 0$
   (B) $x = 1$
   (C) $x = 3$
   (D) $x = 1$ and $x = 3$
   (E) $x = 0$ and $x = 1$ and $x = 3$

8. Assume that $f''(x) = 12x^2 - 6x + 4$. If $f'(1) = 3$ and $f(2) = 7$, find $f(x)$.

   (A) $f(x) = x^4 - x^3 + x^2 + 2x - 9$
   (B) $f(x) = x^4 - x^3 + 2x^2 - x - 7$
   (C) $f(x) = x^4 - x^3 + x^2 - 2x - 1$
   (D) $f(x) = x^4 - x^3 + 2x^2 - 2x - 5$
   (E) None of the above

Record the correct answer to the following problems on the front page of this exam.

9. Let $f(x) = x^3 - 3x + 4$. Find a number $c \geq 0$ that satisfies the conclusions of the Mean Value Theorem over the interval $[-4, 4]$.

   (A) $\frac{4}{\sqrt{3}}$

   (B) 0

   (C) 2

   (D) $\frac{2}{\sqrt{3}}$

   (E) $\sqrt{\frac{13}{3}}$

10. Assume that x, y are positive numbers such that $x + y = 6$. Find the smallest possible value of $2x^2 + 3y^2$.

    (A) 43

    (B) $43\,\frac{1}{5}$

    (C) $43\,\frac{2}{5}$

    (D) $43\,\frac{3}{5}$

    (E) $43\,\frac{4}{5}$

Record the correct answer to the following problems on the front page of this exam.

11. Find the most general antiderivative of $3\sqrt{x} + \sin(x) - \dfrac{2}{x^2}$.

(A) $\frac{2}{3}x^{3/2} - \cos(x) + \frac{2}{x} + C$
(B) $2x^{3/2} + \cos(x) - \frac{2}{x} + C$
(C) $\frac{2}{3}x^{3/2} + \cos(x) + \frac{2}{x} + C$
(D) $2x^{3/2} - \cos(x) - \frac{2}{x} + C$
(E) $2x^{3/2} - \cos(x) + \frac{2}{x} + C$

12. Which of the following definite integrals equals the following expression?

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} \left(\frac{9i^2}{n^2} + \frac{12i}{n} - 5 \right)$$

(A) $\int_0^3 (3x^2 + 12x - 5)\, dx$
(B) $\int_0^3 (9x^2 + 12x - 5)\, dx$
(C) $\int_0^3 (9x^2 + 4x - 5)\, dx$
(D) $\int_0^3 (x^2 + 4x - 5)\, dx$
(E) $\int_0^3 (x^2 + 12x - 5)\, dx$

Free Response Questions: Show your work!

13. Find the limits or state that the limit does not exist. In each case, justify your answer.

    (Students who guess the answer based on a few values of the function will not receive full credit.)

    (a) $\lim_{x\to 0} \dfrac{e^{5x} - 1 - 5x}{3x^2}$

    $\lim_{x\to 0} \dfrac{e^{5x} - 1 - 5x}{3x^2} = \lim_{x\to 0} \dfrac{5e^{5x} - 5}{6x} = \lim_{x\to 0} \dfrac{25e^{5x}}{6} = \dfrac{25}{6}.$

    (b) Find all values of A for which we can use L'Hopital's rule to evaluate $\lim_{x\to 4} \dfrac{x^2 + Ax - 4}{x - 4}$. Then use L'Hopital's rule to evaluate this limit.

    We need $0 = 4^2 + A \cdot 4 - 4 = 0$. Thus $A = -3$. Then $\lim_{x\to 4} \dfrac{x^2 - 3x - 4}{x - 4} = \lim_{x\to 4} \dfrac{2x - 3}{1} = 5$.

14. You may use the following formulas in this problem.

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Recall the formula $R_n = \sum_{i=1}^{n} f(x_i)\Delta x$ where $x_i = a + i\Delta x$ and $\Delta x = \dfrac{b-a}{n}$.

Let $f(x) = x^2 + 2x + 3$. Compute R_n for $f(x)$ over the interval $[0,3]$. (Your final answer should not contain any i's.)

We have $\Delta x = \dfrac{3}{n}$ and $x_i = 0 + i\Delta x = \dfrac{3i}{n}$. $R_n = \left(\sum_{i=1}^{n}\left(\dfrac{3i}{n}\right)^2 + 2\left(\dfrac{3i}{n}\right) + 3\right)\dfrac{3}{n} = \dfrac{27}{n^3}\sum_{i=1}^{n} i^2 + \dfrac{18}{n^2}\sum_{i=1}^{n} i + \dfrac{9}{n}\cdot n = \dfrac{27(n+1)(2n+1)}{6n^2} + \dfrac{18(n+1)}{2n} + 9.$

Free Response Questions: Show your work!

15. A box with a square base and an open top is to have a volume of 64 m^3. Material for the base costs \$54 per square meter and material for the sides costs \$8 per square meter. Find the dimensions of the cheapest box.

    $x^2h = 64$ where x is the length of the side of the base and h is the height. Let C denote the cost. Then $C(x) = 54 \cdot x^2 + 8 \cdot 4xh = 54x^2 + 32x(\frac{64}{x^2}) = 54x^2 + \frac{32 \cdot 64}{x}$. $C'(x) = 108x - \frac{32 \cdot 64}{x^2} = 0$. Then $108x^3 = 32 \cdot 64$, so $x^3 = \frac{8^3}{3^3}$. Thus $x = \frac{8}{3}$. It follows that $h = \dfrac{64}{(8/3)^2} = 9$. The dimensions are $x = \frac{8}{3}$ meters and $h = 9$ meters.

Free Response Questions: Show your work!

16. (To receive credit, you must justify your work.))

    (a) State the Mean Value Theorem.

    Assume that f is a continuous function on an interval $[a, b]$ and differentiable on (a, b). Then there exists a number c in the interval (a, b) such that $\dfrac{f(b) - f(a)}{b - a} = f'(c)$.

    (b) Suppose that f is a differentiable function on the real line and $2 \leq f'(x) \leq 5$ for all x in the interval $(3, 8)$. If $f(3) = 7$, use the Mean Value Theorem for f in the interval $[3, 8]$ to determine the largest and smallest possible values for $f(8)$.

    (Suggestion: Use the Mean Value Theorem to express $f(8)$ in terms of $f(3)$ and $f'(c)$ for some c. Then use the bounds on the derivative that are given.)

    By the Mean Value Theorem, there exists c in $(3, 8)$ such that $\dfrac{f(8) - f(3)}{8 - 3} = f'(c)$. Then $f(8) = f(3) + 5f'(c)$. This gives $f(8) = 7 + 5f'(c)$. Since $f'(c)$ satisfies $2 \leq f'(c) \leq 5$. Thus $17 = 7 + 5 \cdot 2 \leq f(8) = 7 + 5f'(c) \leq 7 + 5 \cdot 5 = 32$. That is, $17 \leq f(8) \leq 32$.