

Name: _____

Section and/or TA: _____

Do not remove this answer page. You will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or communication capabilities is permitted. You may not use a phone or other communication device during the exam.

The exam consists of 12 multiple choice questions that are worth 5 points each and 4 free response questions that are worth 10 points each. You should work the multiple choice questions on the question page. After you have checked your work carefully, record your answers by completely filling in the circle below that corresponds to your answer. If you must change your answer, make a note on the front of the exam. Be sure to check carefully when you transfer your answers to the cover sheet.

Show all work to receive full credit on the free response problems. You do not need to compute a decimal approximation to your answer. For example, the answer 4π is preferred to 12.57.

Multiple Choice Questions

1 A B C D E**7** A B C D E**2** A B C D E**8** A B C D E**3** A B C D E**9** A B C D E**4** A B C D E**10** A B C D E**5** A B C D E**11** A B C D E**6** A B C D E**12** A B C D E

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

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Multiple Choice Questions

1. (5 points) Let

$$f(x) = \begin{cases} 2x + c, & x < 2 \\ x^3 - c, & x \geq 2 \end{cases}$$

For what value(s) of c is this function continuous everywhere?

- A. $c = 0$
 - B. $c = 4$
 - C. $c = -4$
 - D. $c = 2$
 - E. There is no value of c for which f is continuous
2. (5 points) A function f satisfies $x^2 - x + 7 \leq f(x) \leq 2x^2 + x + 8$ for all real numbers x . There is exactly one real number c where we may use the Squeeze Theorem to compute the limit $\lim_{x \rightarrow c} f(x) = L$. Find c and L .

- A. $c = -1$ and $L = 0$
- B. $c = 2$ and $L = 9$
- C. $c = -1$ and $L = 9$
- D. $c = 0$ and $L = 8$
- E. $c = 0$ and $L = 7$

3. (5 points) Find the value of the limit

$$\lim_{x \rightarrow 1} \frac{e^{2x} - e^2}{\ln x}$$

- A. ∞
- B. e^2
- C. 1
- D. $2e^2$
- E. 0

4. (5 points) Consider the curve defined by $y^5 + xy^2 - 3x = 3$. Find the tangent line to this curve at the point $(x, y) = (-1, 1)$.

- A. $y = \frac{1}{2}x + \frac{3}{2}$
- B. $y = \frac{2}{3}x - \frac{5}{3}$
- C. $y = \frac{1}{2}x + \frac{3}{2}$
- D. $y = -\frac{2}{3}x + \frac{5}{3}$
- E. $y = \frac{2}{3}x + \frac{5}{3}$

5. (5 points) Find the quadratic approximation of $f(x) = \cos(2x)$ at $a = 0$.

A. $1 + 2x^2$

B. $1 + x^2$

C. $-x^2$

D. $1 - x^2$

E. $1 - 2x^2$

6. (5 points) Let $f(x) = \frac{x^2}{x-4}$. What are all the local minima of f ?

A. $x = 8$

B. $x = 4$

C. $x = 0$

D. f does not have any local minima.

E. $x = 0$ and $x = 8$

7. (5 points) Consider the integral $\int_5^8 e^{x^2} dx$. Give the value of the right Riemann sum formed using three equal subintervals and the right endpoint of each subinterval as the sample points.

- A. 67
- B. $e^{25} + e^{36} + e^{49}$
- C. $e^{36} + e^{49} + e^{64}$
- D. $\frac{1}{3}(e^{25} + e^{36} + e^{49})$
- E. $e^6 + e^7 + e^8$

8. (5 points) Determine which of the following integrals is equal to the given limit without evaluating the limit:

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{6}{n} \arctan\left(5 + \frac{3i}{n}\right)$$

- A. $\int_6^7 2 \arctan(x) dx$
- B. $\int_5^8 2 \arctan(x) dx$
- C. $\int_6^7 \arctan(x) dx$
- D. $\int_5^8 \arctan(x) dx$
- E. $\int_5^{11} 2 \arctan(x) dx$

9. (5 points) Use L'Hôpital's rule and the Fundamental Theorem of Calculus to evaluate the following limit

$$\lim_{x \rightarrow 0} \frac{\int_0^x (e^{3t} - 1) dt}{x^2}$$

- A. 1
B. $3/2$
C. 3
D. 0
E. The limit does not exist
10. (5 points) Assuming that $\int_6^7 f(x) dx = 3$ and $\int_5^7 f(x) dx = 2$, find $\int_5^6 2f(x) dx$.

- A. 2
B. 1
C. -2
D. -1
E. 0

11. (5 points) Use the Fundamental Theorem of Calculus to find the function f that verifies the following equation

$$\int_0^x \frac{f(t)}{1+t^2} dt = \ln(1+x).$$

- A. $f(x) = \arctan x$
- B. $f(x) = (1+x^2)\ln(1+x)$
- C. $f(x) = \frac{1+x^2}{1+x}$
- D. $f(x) = x^2$
- E. $f(x) = e^x$

12. (5 points) Evaluate the indefinite integral $\int \frac{x^3}{1+x^4} dx$.

- A. $\frac{1}{4}\ln(1+x^4) + C$
- B. $\arctan x + C$
- C. $\arctan(1+x^4) + C$
- D. $\frac{1}{4}\arctan x + C$
- E. $\ln(1+x^4) + C$

Free response questions: Show work clearly with proper notation.

13. (10 points) Compute the derivatives below

(a) $\frac{d}{dx}(e^{67x} \arctan x)$

(b) $\frac{d}{dx} \left(\frac{2x+3}{x^2+10} \right)$

(c) $\frac{d}{dx} \int_0^{5x} \sqrt{t^2 + e^{-t}} dt$

Solution: a) Use the product rule and chain rule to find

$$\frac{d}{dx}(e^{67x} \arctan x) = (e^{67x})' \arctan x + e^{67x} (\arctan x)' = 67e^{67x} \arctan x + \frac{e^{67x}}{1+x^2}$$

b) Use the quotient rule to write

$$\begin{aligned} \frac{d}{dx} \left(\frac{2x+3}{x^2+10} \right) &= \frac{(2x+3)'(x^2+10) - (x^2+10)'(2x+3)}{(x^2+10)^2} \\ &= \frac{2(x^2+10) - (2x)(2x+3)}{(x^2+10)^2} \\ &= \frac{-2x^2 - 6x + 20}{(x^2+10)^2} \end{aligned}$$

c) We use FTC I and the chain rule to find

$$\frac{d}{dx} \int_0^{5x} \sqrt{t^2 + e^{-t}} dt = \sqrt{(5x)^2 + e^{-5x}} (5x)' = 5\sqrt{25x^2 + e^{-5x}}.$$

Grading:

a) Product rule (2 point), chain rule (1 point), answer (1 point).

b) Quotient rule (2 points), answer (1 point)

c) Recognizing the function inside the integral as the derivative (1 point), evaluating at $5x$ (1 point), answer (1 point)

14. (10 points) Let $f(x) = \int_0^x \frac{(42t - t^2)}{e^{2t+4} + 1} dt$.

- Find f' .
- Find the intervals where f is increasing and decreasing.
- Find each value x where f has a local maximum and each value x where f has a local minimum. Explain how you determine if each is a local maximum or minimum.

Solution: WW II-1.3 #10

a) According to FTC I, the derivative of f , $f'(x) = \frac{(42x - x^2)}{e^{2x+4} + 1} = \frac{(42 - x)x}{e^{2x+4} + 1}$. We find the critical numbers by solving $f'(x) = 0$. Since, $e^{2x+4} + 1 > 0$, $f'(x) = 0$ implies that $(42 - x)x = 0$. Thus, the critical numbers are 0 and 42.

Next, we check the signs of $f'(x)$ on the intervals $(-\infty, 0)$, $(0, 42)$, and $(42, \infty)$. Note that the sign of $f'(x)$ is determined by its numerator because the denominator is always positive.

Thus,

- f is increasing in $(0, 42)$ and
- decreasing in $(-\infty, 0)$ and $(42, \infty)$.

c) There is a local minimum at $x = 0$ and a local maximum at $x = 42$. One can use first or second derivative test to justify

Grading:

- Find f' using FTC I (3 points)
- Critical points (2 points) Interval of increase and decrease (2 points)
- Locations of local maxima or minima (2 points) Justification (1 point)

15. (10 points) Evaluate the following integrals using substitution. You must clearly show steps of the substitution to receive full credit.

(a) $\int_0^1 x\sqrt{x^2+1} dx$

(b) $\int \sin x \cos x dx$

Solution:

a) Set $u = x^2 + 1$, then $\frac{du}{dx} = 2x$. The new bounds of integration are 1 and 2. Then

$$\int_0^1 x\sqrt{x^2+1} dx = \int_1^2 \frac{1}{2}\sqrt{u} du = \frac{1}{3}u^{3/2}\Big|_1^2 = \frac{1}{3}(2^{3/2} - 1)$$

b) Set $u = \sin(x)$, then $du = \cos x dx$ and we have

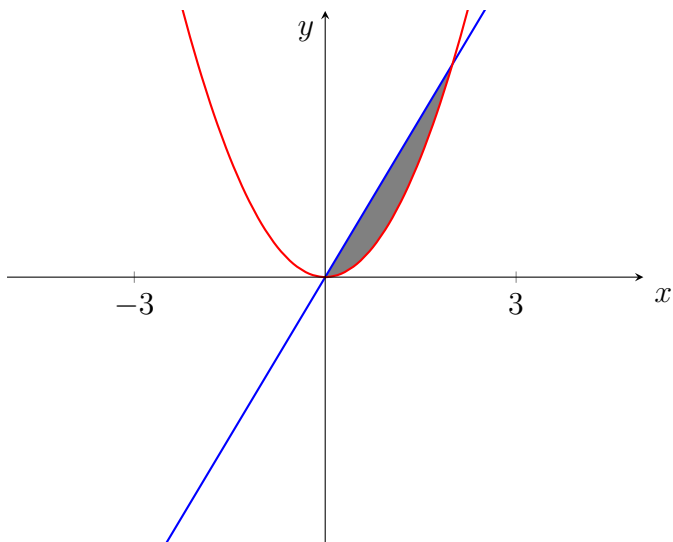
$$\int \sin x \cos x dx = \int u du = \frac{1}{2}u^2 + C = \frac{1}{2}(\sin x)^2 + C$$

Grading:

a) Set the correct substitution (2 points), bounds of integration (1 point), find antiderivative in terms of u (1 point), final answer (1 point)

b) Set the correct substitution (2 points), simplify and find antiderivative in terms of u (2 points), write answer in terms of x (1 point)

16. (10 points) Below are the graphs of $f(x) = x^2$ and $g(x) = 2x$.
- (a) Set up an integral whose value is the area of the shaded region.
- (b) Evaluate your integral to find the area of the region.

**Solution:**

The graphs of x^2 and $2x$ intersect for x where $x^2 = 2x$. The solutions are $x = 0, 2$.
The area between the graphs of $y = x^2$ and $y = 2x$ is

$$\int_0^2 2x - x^2 dx.$$

We use FTC II to evaluate the integral

$$\begin{aligned} \int_0^2 2x - x^2 dx &= \left(x^2 - \frac{x^3}{3} \right) \Big|_{x=0}^2 \\ &= 4 - \frac{8}{3} = \frac{4}{3}. \end{aligned}$$

Grading:

Points of intersection (2 points), expression of area as an integral (3 points), finding anti-derivative (3 points), value for area (2 points).