Name: ______________________ Section and/or TA: ______________

Do not remove this answer page. You will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or communication capabilities is permitted. You may not use a phone or other communication device during the exam.

The exam consists of 12 multiple choice questions that are worth 5 points each and 4 free response questions that are worth 10 points each. You should work the multiple choice questions on the question page. After you have checked your work carefully, record your answers by completely filling in the circle below that corresponds to your answer. If you must change your answer, make a note on the front of the exam. Be sure to check carefully when you transfer your answers to the cover sheet.

Show all work to receive full credit on the free response problems. You do not need to compute a decimal approximation to your answer. For example, the answer 4π is preferred to 12.57.

Multiple Choice Questions

1 (A) (B) (C) (D) (E) **7** (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E) **8** (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E) **9** (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E) **10** (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E) **11** (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E) **12** (A) (B) (C) (D) (E)

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

This page is left blank.

Multiple Choice Questions

1. (5 points) Let

$$f(x) = \begin{cases} -x^2 + 5x - c & x < 4 \\ x^2 - cx + 21 & x \geq 4 \end{cases}$$

For what value(s) of c is this function continuous?

A. $c = 4$

B. $c = -7$

C. $c = -4$

D. $c = 11$

E. There is no value of c for which f is continuous

2. (5 points) A function f satisfies $-x^2+x+17 \leq f(x) \leq x^2+9x+25$ for all real numbers x. There is exactly one real number c where we may use the Squeeze Theorem to compute the limit $\lim_{x \to c} f(x) = L$. Find c and L.

A. $c = -1$ and $L = 15$

B. $c = 2$ and $L = 15$

C. $c = -2$ *and* $L = 11$

D. $c = -2$ and $L = 4$

E. $c = 0$ and $L = 25$

3. (5 points) Consider the limit $\lim_{h \to 0} \frac{(2+h)^2 - 2^2}{h} = L$. Select the correct statement.

   A. The value L is the derivative of $x + 2$ at $x = 0$.

   B. The value L is the derivative of $x + h$ at $x = 0$.

   C. The value L is the derivative of $(x + 2)^2$ at $x = 2$.

   D. The value L is the derivative of x^2 at $x = 2$.

   E. The value L is the derivative of $x + 2$ at $x = 2$.

4. (5 points) Consider the ellipse defined by $x^2 - xy + 2y^2 = 4$. Find the tangent line to this curve at the point $(x, y) = (-2, 0)$.

   A. $y = 2x - 2$

   B. $y = 2x - 4$

   C. $y = -2x - 4$

   D. $y = -2x + 4$

   E. $y = 2x + 4$

5. (5 points) The height (in meters) of a bullet fired in the air vertically from ground level is $s(t) = -3t^2 + 12t + 1$. Find the bullet's maximum height.

   A. 36 meters

   B. 72 meters

   C. 2 meters

   D. 13 meters

   E. 12 meters

6. (5 points) Let $f(x) = \ln(x^2 + 1)$. On what intervals is f concave up?

   A. f is concave up on $(-1, 1)$

   B. f is concave up on $(1, +\infty)$

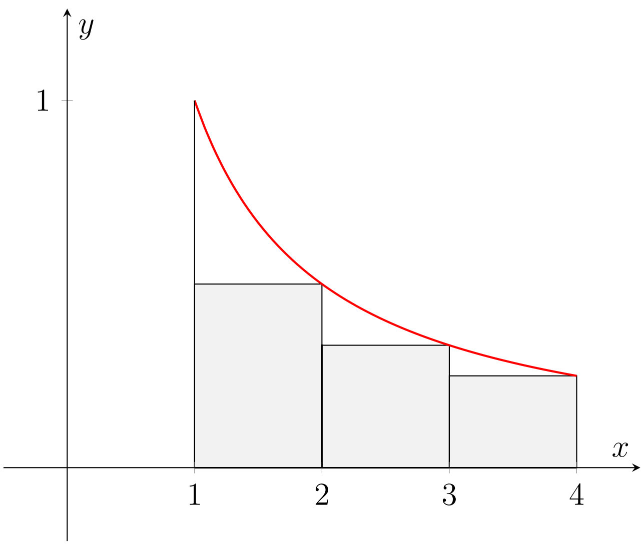
   C. f is concave up on $(0, +\infty)$

   D. f is concave up on $(-\infty, -1)$

   E. f is concave up on $(-\infty, 0)$

7. (5 points) Consider the function $f(x) = \dfrac{1}{x}$ defined on the interval $[1, 4]$.

Estimate $\displaystyle\int_1^4 \frac{1}{x}\,dx$ using right endpoints for $n = 3$ approximating rectangles all having bases of the same length, as shown in the picture.



A. 1/2 B. 11/6 *C. 13/12* D. ln(4) E. 5/6

8. (5 points) Determine which of the following integrals is equal to the given limit without evaluating the limit:

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{8}{n}\cos\left(2+\frac{4i}{n}\right)$$

A. $\displaystyle\int_4^8 \cos(x)\,dx$

B. $\displaystyle\int_2^6 2\cos(x)\,dx$

C. $\displaystyle\int_2^4 \cos(x)\,dx$

D. $\displaystyle\int_2^4 8\cos(x)\,dx$

E. $\displaystyle\int_2^6 \cos(x)\,dx$

9. (5 points) Use L'Hôpital's rule and the Fundamental Theorem of Calculus to evaluate the following limit

$$\lim_{x\to 0} \frac{\displaystyle\int_0^x (18 - 18\cos(t))\, dt}{x^3}$$

A. 1

B. 3

C. 7/6

D. 0

E. The limit does not exist

10. (5 points) Assuming that $\displaystyle\int_2^5 f(x)\ dx = \frac{1}{2}$ and $\displaystyle\int_7^2 f(x)\ dx = \frac{3}{2}$, find $\displaystyle\int_5^7 f(x)\ dx$.

A. 1

B. 5/2

C. -2

D. 1/2

E. 2

11. (5 points) Use the Fundamental Theorem of Calculus to find the function f that verifies the following equation

$$\int_{\frac{\pi}{2}}^{x} \frac{f(t)}{\sin(t)}\, dt = -\cos(x)$$

A. $f(x) = \dfrac{(\cos(x))^2}{2}$

B. $f(x) = \cos^2(x)$

C. $f(x) = \sin^2(x)$

D. $f(x) = -\sin(x)\cos(x)$

E. $f(x) = \ln(\sin(x))$

12. (5 points) An object is moving along a line so that its velocity at time t is $v(t) = -3t^2 + 2$ meters/second. Find the change in position between $t = 0$ and $t = 5$ seconds. Assume that displacement to the right is positive.

A. 115 meters to the left

B. 73 meters to the left

C. 5 meters to the left

D. 73 meters to the right

E. 5 meters to the right

Free response questions: Show work clearly with proper notation.

13. (10 points) Compute the derivatives, you do not need to simplify your answers

(a) $\frac{d}{dx}(\sin(x)\ln(x^2))$

(b) $\frac{d}{dx}\left(\frac{x^2+5}{x^2+7}\right)$

(c) $\frac{d}{dt}\sqrt{e^{5t}+1}$

Solution: a) Use the product rule and chain rule to find

$$\frac{d}{dx}(\sin(x)\ln(x^2)) = \cos(x)\ln(x^2) + \sin(x)\cdot\frac{1}{x^2}\cdot\frac{d}{dx}(x^2) = \cos(x)\ln(x^2) + \sin(x)\cdot\frac{2x}{x^2}$$

b) Use the quotient rule to write

$$\begin{aligned}\frac{d}{dx}\left(\frac{x^2+5}{x^2+7}\right) &= = \frac{(x^2+5)'(x^2+7)-(x^2+5)(x^2+7)'}{(x^2+7)^2}\\ &= \frac{(2x)(x^2+7)-(2x)(x^2+5)}{(x^2+7)^2}\\ &= \frac{14x}{(x^2+7)^2}\end{aligned}$$

c) We write the radical as a power and use the chain rule to find

$$\frac{d}{dx}(e^{5t}+1)^{1/2} = \frac{1}{2}(e^{5t}+1)^{-1/2}\cdot 5(e^{5t}) = \frac{5e^{5t}}{2\sqrt{e^{5t}+1}}.$$

Grading:

a) Product rule (2 point), chain rule (1 point), answer (1 point).

b) Quotient rule (2 points), answer (1 point)

c) Derivative of square root (1 point), chain rule (1 point), answer (1 point)

Notes: Problem does not ask for simplification. A student who applies a rule, but with some minor error may receive the credit for the rule, but not for the correct answer.

14. (10 points) Let $f(x) = \int_0^x \frac{(36 - t^2)}{\cos^2(t) + 1}\, dt$.

(a) Find f'.

(b) Find the intervals where f is increasing and decreasing.

(c) Find each value x where f has a local maximum and each value x where f has a local minimum. Explain how you determine if each is a local maximum or minimum.

Solution: WW II-1.3 #10

a) According to FTC I, the derivative of f, $f'(x) = \frac{36 - x^2}{\cos^2(x) + 1} = \frac{(6 - x)(6 + x)}{\cos^2(x) + 1}$.

We find the critical numbers by solving $f'(x) = 0$. Since, $\cos^2(x) + 1 > 0$, $f'(x) = 0$ implies that $(6 - x)(6 + x) = 0$. Thus, the critical numbers are -6 and 6.

Next, we check the signs of $f'(x)$ on the intervals $(-\infty, -6)$, $(-6, 6)$, and $(6, \infty)$. Note that the sign of $f'(x)$ is determined by its numerator because the denominator is always positive.

Thus,

- f is increasing in $(-6, 6)$ and
- decreasing in $(-\infty, -6)$ and $(6, \infty)$.

c) There is a local minimum at $x = -6$ and a local maximum at $x = 6$.

Grading:

a) Find f' using FTC I (3 points)

b) Interval of increase and decrease (4 points)

c) Locations of local maxima or minima (3 points)

15. (10 points) Evaluate the following integrals using substitution. You must clearly show steps of the substitution to receive full credit.

(a) $\displaystyle\int \frac{\cos(7x)}{\sin(7x)}\,dx$

(b) $\displaystyle\int \frac{4}{x}\ln(x)\,dx$

Solution:

a) Set $u = \sin(7x)$, then $\frac{du}{dx} = 7\cos(7x)$. Then

$$\int \frac{\cos(7x)}{\sin(7x)}\,dx = \frac{1}{7}\int \frac{1}{u}\,du = \frac{\ln(\mid u \mid)}{7} + C = \frac{\ln(\mid \sin(7x) \mid)}{7} + C$$

b) Set $u = \ln(x)$, then

$$\int \frac{4}{x}\ln(x)\,dx = 4\int u\,du = 4(\frac{1}{2})u^2 + C = 2\ln(\mid x \mid)^2 + C$$

Grading:

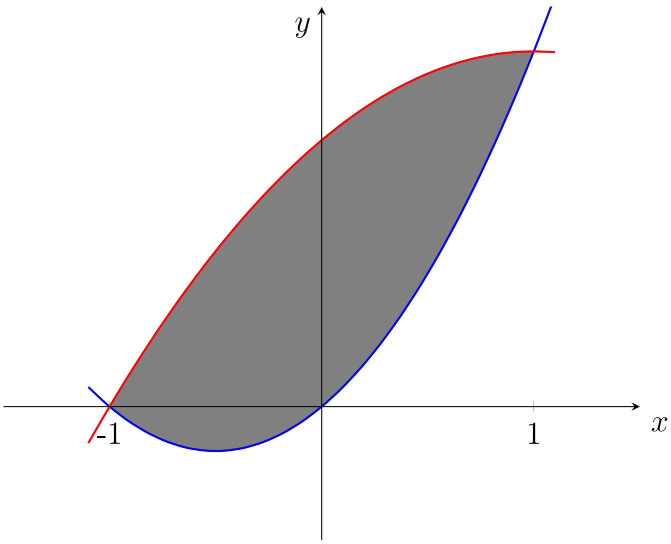
a) Set the correct substitution (2 points), simplify and find antiderivative in terms of u (2 point), write answer in terms of x (1 point)

b) Set the correct substitution (2 points), simplify and find antiderivative in terms of u (2 point), write answer in terms of x (1 point)

16. (10 points) Below are the graphs of $f(x) = -x^2 + 2x + 3$ and $g(x) = 2x^2 + 2x$.

(a) Set up an integral whose value is the area of the shaded region.

(b) Evaluate your integral to find the area of the region.



Solution:

The graphs of $-x^2+2x+3$ and $2x^2+2x$ intersect for x where $-x^2+2x+3 = 2x^2+2x$ or $x^2 - 1 = 0$. The solutions are $x = -1, 1$.

The area between the graphs of $y = -x^2 + 2x + 2$ and $y = x^2 + 2x$ is

$$\int_{-1}^{1} [-x^2 + 2x + 3 - (2x^2 + 2x)]\, dx = \int_{-1}^{1} [3 - 3x^2]\, dx.$$

We use FTC II to evaluate the integrals,

$$\begin{aligned} \int_{-1}^{1} [3 - 3x^2]\, dx &= (3x - \frac{3x^3}{3})\Big|_{x=-1}^{1} \\ &= (3 - (1)^3) - (-3 - (-1)^3) = 4. \end{aligned}$$

Grading:

Points of intersection (2 points) (may read from graph), expression of area as an integral (3 points), finding anti-derivative (3 points), value for area (2 points).

Students may read points of intersection off of graph. Students who compute the definite integral without computing the antiderivatives should receive half credit.