MA 113 **Name:** _____

Quiz 10 - December 5, 2013

1. Evaluate the following integrals:

   (a) $\displaystyle\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}}dx$

   Solution: Let $u = \sqrt{x}$. Then, $du = \frac{1}{2} \cdot \frac{1}{\sqrt{x}}dx$ or rather $2du = \frac{1}{\sqrt{x}}dx$. Our equation then becomes

   $$\begin{aligned}\int \sec^2(u)2du &= 2\int \sec^2(u)du \\ &= 2\tan(u) + C\end{aligned}$$

   Substituting $u = \sqrt{x}$ gives the final answer:

   $$\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}}dx = 2\tan(\sqrt{x}) + C$$

   (b) $\displaystyle\int_0^3 \left|x^2 - 4x + 3\right| dx$

   Solution: First factor $|x^2 - 4x + 3|$ as $|(x-3)(x-1)|$. Then, $(x-3)(x-1) > 0$ when $x \in (-\infty, 1)$ or $x \in (3, \infty)$. Furthermore, $(x-3)(x-1) < 0$ when $1 < x < 3$. So,

   $$\begin{aligned}\int_0^3 \left|x^2 - 4x + 3\right| dx &= \int_0^3 \left|x^2 - 4x + 3\right| dx \\ &= \int_0^1 \left|x^2 - 4x + 3\right| dx + \int_1^3 \left|x^2 - 4x + 3\right| dx \\ &= \int_0^1 x^2 - 4x + 3dx + \int_1^3 -(x^2 - 4x + 3)dx \\ &= \int_0^1 x^2 - 4x + 3dx + -\int_1^3 (x^2 - 4x + 3)dx \\ &= \left[\frac{x^3}{3} - 2x^2 + 3x\right]_{x=0}^{x=1} - \left[\frac{x^3}{3} - 2x^2 + 3x\right]_{x=1}^{x=3} \\ &= \left(\frac{1}{3} - 2 + 3\right) - 0 - \left(\left(\frac{27}{3} - 18 + 9\right) - \left(\frac{1}{3} - 2 + 3\right)\right) \\ &= \frac{8}{3}\end{aligned}$$

2. John buys his children, Joshua and Raphael, each a toy called a Mogwai. The children are told to never spill water on the toys. The children, however, are reckless and spill water on both toys. The children start with two Mogwai at time $t = 0$. However, in a minute, at time $t = 1$, there is a total of 6 toys. Let $P = P(t)$ be the population of the

toys at time t where t is measured in minutes. Assume that the growth of the toys is continuous and that once one Mogwai is wet, all future Mogwai are wet as well.

(a) Since the rate of increase of the population is proportional to the current population, then $P = Ce^{kt}$. Find C and k.

Solution: At time $t = 0, P = 2$. So,

$$2 = Ce^{k \cdot 0} = C$$

So $C = 2$. To find k, evaluate at $t = 1$.

$$\begin{aligned} 6 &= 2 \cdot e^{k \cdot 1} \\ 3 &= e^{k} \\ \ln(3) &= k \end{aligned}$$

So $k = \ln(3)$

(b) If the father comes into the room at time $t = 10$, approximately how many Mogwai will there be?

Solution: From above, $P = e^{ln(3)t}$. Evaluating at $t = 10$ yields

$$\begin{aligned} P &= 2 \cdot e^{\ln(3) \cdot 10} \\ &= 118,098 \end{aligned}$$