

1. Suppose $f(x) = x^2 + \sqrt{27}x + 6$ and $g(x) = \sqrt{3}x$. Write $(f \circ g)(x)$ in the form $ax^2 + bx + c$ and find the roots of $(f \circ g)(x)$.

Solution:

$$(f \circ g)(x) = f(g(x)) = (\sqrt{3}x)^2 + \sqrt{27}(\sqrt{3}x) + 6 = 3x^2 + 9x + 6$$

Then set $(f \circ g)(x) = 0$ to find the roots.

$$3x^2 + 9x + 6 = 3(x^2 + 3x + 2) = 0$$

Factoring we see that $3(x+2)(x+1) = 0$, so the roots are $x = -2$ and $x = -1$.

2. Find the inverse defined by $f(x) = 2^{2x+3}$. What are the domain and range of f^{-1} ?

Solution:

Start with the equation $y = 2^{2x+3}$

Apply \log_2 to both sides to get $\log_2(y) = 2x + 3$ or $\frac{\ln(y)}{\ln(2)} = 2x + 3$

Subtract 3 and divide by 2 to get $\frac{\ln(y)}{2\ln(2)} - \frac{3}{2} = x$

This gives $f^{-1}(x) = \frac{\ln(x)}{\ln(4)} - \frac{3}{2}$

The domain of \ln is $(0, \infty)$, and the range of \ln is $(-\infty, \infty)$. Therefore, the domain of $f^{-1}(x)$ is $(0, \infty)$ and the range is $(-\infty, \infty)$.